APPLICATIONS OF HYDROLOGY TO WATER RESOURCES MANAGEMENT

(Planning and design level)

by V. Klemes

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FOREWORD

The application of hydrological techniques to the planning and design of water resources projects has always been of direct concern to hydrologists who, in view of the increasingly complex scope of water resources projects, need to have a sound knowledge of the classical as well as the most modern analytical techniques applied in this field of operational hydrology. For this purpose, the WMO Commission for Hydrology included in the WMO Guide to Hydrological Practices an annex containing an outline of such techniques.

In order to provide additional guidance material on this subject, the WMO Secretariat, at the request of the Commission for Hydrology, arranged for the preparation of this report by Dr. V. Klemeš (Canada). The note was reviewed by the Commission's Working Group on Design Data for Water Resources Projects, which suggested that it be published by WMO in its Operational Hydrology Reports series of publications.

It is a great pleasure for me to express the thanks of WMO to Dr. Klemeš for his valuable contribution to a very complex and important topic.

D. A. Davies
Secretary-General
The present report is not intended as a substitute for a textbook or a handbook on applied hydrology. The focus is not on presenting a complete survey of hydrological problems that are of interest to water resources management. Rather, its aim is to show some of these problems in a somewhat unorthodox perspective; in the perspective of a water resources manager who finds himself in a region that is to be economically developed and who is supposed to introduce a rational system of management of the local water resources. The question is: What does he need to know about the water resources he is supposed to manage, and how can hydrology help him? Obviously, he needs information on the quantity and quality of water available and he also needs information on the water needs. If there is either no water or no water needs, then there is no water management, for in the author's interpretation the latter represents a decision-making process concerning the distribution of water among water users.

Hydrology can provide the water resources manager with information on water quantity. Based on historical records of various hydrological phenomena, and on analysis of the processes of the hydrological cycle, the hydrologist can make a more or less accurate quantitative assessment of the distribution of water in space and time, and can quantitatively evaluate the impact of man-made changes in the natural water distribution pattern.

The manager needs, in principle, two different kinds of hydrological information. In the first place, he needs information on the basis of which he could make fundamental long-term, and very often irreversible, decisions. For instance, whether to build a dam or not; if so, how big it should be; what capacity its outlets should have. Secondly, he needs information for operational, day-to-day and often hour-to-hour, water management. For instance, how best to operate a dam in case of a particular flood, how to minimize water shortages in a particular dry season, etc.

The present report deals only with the first type of hydrological problems, with problems encountered mainly in the planning and design of water resources projects. Problems of the second category, not discussed herein, often require different approaches and assign different degrees of importance to various phenomena than do the problems of the first category. For instance, when designing a dam that is supposed to control a high portion of a basin's runoff (e.g., the high Aswan dam) the sequence of mean annual flows will be of paramount importance while the day-to-day flow forecasting will be irrelevant. However, once the dam has been built the reverse is true: its operation will depend very much on the short-term forecasting and very little on the sequencing of mean annual flows.

Another limitation of the report consists in the fact that it deals with surface water only, leaving the entire domain of groundwater out of the picture. This should by no means be interpreted as if groundwater problems were less important than those concerning surface water; on the contrary, often the reverse is true. The reason for leaving out the groundwater-related problems is the lack of the author's experience and expertise in this area.
Finally, a few words should be said regarding the method used in presenting the material. It is assumed that the reader is familiar with the basics of surface water hydrology including some classical concepts such as the unit hydrograph, regression analysis, etc. Some concepts are therefore often referred to without detailed explanations which are included only if it is considered helpful in a particular context. For instance, many types of probability distributions are referred to without the analytical form of a single one being given. On the other hand, the matrix form for a convolution is given twice, each time in a different context. The reason is that in the first case the emphasis is not on the mathematical form at all while the second case is to demonstrate how different hydrological concepts can lead to the same mathematical formulation. In general, the emphasis throughout the report is on the arrangement of the material, on its exposure in a particular hydrological context, not on the completeness in the coverage. The report is an attempt to view applied hydrology through the eyes of one type of the customers for its end-products - the planner and designer of water resources schemes. This is reflected in the formal structure of the report which tries to follow the basic sequence of steps implied in a water management decision at the planning level. First of all, the decision-maker should find out how much water there is available. Then, to put it simply, he compares the available quantity with the existing and future needs, and ends up with facing one of two possible situations: either there is not enough water or there is a water surplus. Thus there are three instances where he needs the applied hydrologist's help:

(a) In making the water resource inventory;
(b) In coping with problems arising from water scarcity;
(c) In coping with problems related to water excess.

It should also be mentioned that the report, being a personal viewpoint rather than a state-of-the-art paper, may not mention some concepts which are as important as those described. Similarly, although many authors have been cited in the text, there are many more important names that have not been mentioned.

Finally, the author would like to extend his thanks to Dr. Leo R. Beard, Technical Director of the Center for Research in Water Resources at the University of Texas at Austin, for his comments and criticism which were most helpful in the final editing of the report. Naturally, the responsibility for the views expressed therein, as well as for any errors, rests exclusively with the author.

V. Klemeš
SUMMARY

This report discusses the different methods and concepts involved in the practical applications of hydrology and the effective use of hydrological techniques in the planning, design and management of projects in the field of surface water resources.

A general discussion of water resources management is presented in Chapter 1. The new trends, conceptual changes and the gradual broadening in the scope of this discipline, involving the adoption of new methods of analysis (systems analysis, statistical and probabilistic concepts) and the extensive use of high-speed digital computers, are discussed.

One of the applications of hydrology to water resources management includes the quantitative assessment of the available water resources in a so-called water resources inventory. The hydrological problems involved in a surface runoff inventory are discussed in Chapter 2. Techniques of extracting the maximum amount of information from records obtainable from a single station and methods of assessing its reliability are presented. Characteristics of annual runoff are discussed, including the mean, the coefficient of variation, the probability distribution (properties of runoff distributions, distribution models for annual runoff, fitting of distribution models and plotting of distribution functions), the pattern of time fluctuations and methods of simulation of series of annual runoff (simulation of random sequence and first-order Markov chain). Sub-annual fluctuations of runoff, described by the mean monthly and mean daily flows, are discussed and relevant methods of analysis for the simulation of monthly flow series are explained, including the use of the duration curve for the mean daily flows. The use of statistical methods (formulae and regression models) and deterministic models (lumped-system models and distributed-system models) for extending the streamflow records of a single station are fully discussed and, for the statistical approach, the effect of correlation on effectiveness of record extension and the pitfalls of regressions are reviewed. Indirect determination of streamflow from ungauged basins can be made, using the hydrological analogy technique (the concept of the design and analogue basins) and also on the basis of regional characteristics of runoff which include the estimation of specific runoff, coefficient of runoff, regional coefficient of variation of annual runoff and the estimation of the probability distribution of the combined annual or seasonal runoff of two or more rivers in a certain region.

The basic role of hydrology in water resources management relates to the process of matching the available supplies with the needs and involves problems arising from water deficiency on the one hand, and from water excess (floods) on the other. The hydrological problems arising from water deficiency are discussed in Chapter 3. The various interpretations of droughts are reviewed and the objectives of analysis (drought duration, probability of occurrence, severity, time of occurrence and areal extent) and the methods of analysis (empirical methods, Monte Carlo methods and analytical methods) are discussed. The term water deficiency is explained in so far as water needs, demands and requirements, water use, water losses and deficits are
concerned. A detailed discussion of streamflow regulation by means of storage reservoirs is presented, including definitions and features, elements dealt with in streamflow control analysis and their characteristics and types of streamflow regulation. The storage equation (a mathematical relationship between reservoir storage, draft and dependability) is described with a discussion of the characteristics of dependability and target draft. The methods of solving the storage equation can be either deterministic or stochastic. The deterministic methods (numerical and graphical techniques), the stochastic methods (simulation, analytical and numerical techniques) and the semi-stochastic methods of solving the storage equation are discussed. Pre-computed solutions of the storage equation and the effect of time horizon on low-flow regulation are reviewed.

Techniques of flood analysis and hydrological problems caused by floods are described in Chapter 4. An analytical study is presented, involving the definition of a flood, its features (flood elevation, discharge, volume, duration, seasonal occurrence and flow velocity) and the objectives of flood analysis involving the assessment of the degree and frequency of interference of floods with normal life and activities of a community. The methods of defining a flood frequency (methods based on annual maxima and partial duration series) and the methods and problems involved in modelling and estimating the flood frequency distribution are discussed. A detailed discussion is given of the techniques and methods of flood synthesis which includes indirect determination of flood flows using the rational formula and other regional empirical formulae, methods of synthesis of flood hydrographs (geometric methods, correlation methods, equal probability methods and the maximum probable flood method), flood routing techniques for approximate solution of unsteady flow of water in river channels and through storage reservoirs (reservoir routing and channel routing) and flood control techniques involving controlled and uncontrolled reservoir storage, design of a flood-control reservoir and flood control by a multi-purpose reservoir.
RESUME

Le présent rapport analyse les différentes méthodes et principes auxquels font appel les applications pratiques de l'hydrologie ainsi que l'utilisation judicieuse des techniques de l'hydrologie pour la planification, la conception et la mise en oeuvre des projets concernant l'exploitation des ressources en eaux de surface.

Le chapitre 1 brosse un tableau général de l'exploitation des ressources en eau. Il analyse les nouvelles tendances, l'évolution des concepts et l'élargissement progressif du champ d'action de cette discipline, qui ont conduit à adopter de nouvelles méthodes d'analyse (analyse de systèmes, concepts statistiques et probabilistiques) et à utiliser largement les ordinateurs numériques rapides.

L'une des applications de l'hydrologie à l'exploitation des ressources en eau consiste à dresser ce qu'il est convenu d'appeler un inventaire des ressources en eau, c'est-à-dire évaluer quantitativement les ressources en eau disponibles. Le chapitre 2 traite précisément des problèmes hydrologiques que pose l'établissement d'un inventaire de l'écoulement de surface. Il expose les méthodes à utiliser pour extraire le maximum de renseignements des relevés d'une seule station et pour évaluer la qualité de ces renseignements. Il étudie les caractéristiques de l'écoulement annuel, notamment l'écoulement moyen, le coefficient de variation, la distribution de probabilité (propriétés des distributions de l'écoulement, modèles de distribution de l'écoulement annuel, ajustement des modèles de distribution et pointage des fonctions de distribution), l'allure des fluctuations temporelles et les méthodes de simulation des séries de l'écoulement annuel (simulation d'une suite aléatoire et d'une chaîne de Markov du premier degré). Le chapitre considère ensuite les fluctuations de l'écoulement qui interviennent en cours d'année et qui sont mises en évidence par les moyennes mensuelles et journalières du débit et explique les méthodes d'analyse pertinentes pour simuler les séries de données sur le débit mensuel, notamment comment il convient d'utiliser la courbe de durée pour les débits moyens quotidiens. Il analyse en détail l'utilisation des méthodes statistiques (formules et modèles de régression) et des modèles déterministes (modèles à systèmes groupés et modèles à systèmes répartis) pour étendre les relevés du débit d'une seule station à une période plus longue que celle à laquelle se réfèrent les relevés en insistant, dans le cas des méthodes statistiques, sur l'importance de la corrélation quant à l'efficacité de la reconstitution et sur les pièges de la régression. On peut déterminer indirectement l'écoulement de bassins dans lesquels il n'existe pas de stations de jaugeage, en appliquant la méthode de l'analogie hydrologique (principe du bassin témoin et de bassins analogues), ainsi qu'à partir des caractéristiques régionales de l'écoulement ce qui nécessite d'estimer, d'une part, l'écoulement spécifique, le coefficient d'écoulement, le coefficient régional des variations de l'écoulement annuel et, d'autre part, la distribution de probabilité des débits combinés annuels ou saisonniers d'au moins deux rivières situées dans une région donnée.
En matière d'exploitation des ressources en eau, l'hydrologie intervient de manière fondamentale dans le processus qui consiste à équilibrer au mieux les ressources en eau disponibles et les besoins, c'est-à-dire résoudre les problèmes résultant d'un manque d'eau ainsi que d'un excès d'eau (inondations). Les problèmes hydrologiques associés au manque d'eau sont analysés au chapitre 3. Les diverses interprétations des sécheresses y sont examinées, puis les objectifs de l'analyse (durée de la sécheresse, probabilité d'apparition, rigueur, époque d'apparition, étendue de la zone touchée) et les méthodes d'analyse (méthodes empiriques, méthodes de Monte Carlo et méthodes analytiques) y sont étudiées. L'expression "pénurie d'eau" est expliquée en faisant intervenir les notions de besoins et de demande en eau, d'utilisation de l'eau, de pertes en eau et de déficit en eau. On trouve ensuite une étude détaillée de la régulation de l'écoulement fluvial au moyen de réservoirs d'accumulation qui définit d'abord ce qu'est la régulation et les caractéristiques que doivent posséder les réservoirs, avant d'examiner les éléments qui interviennent dans l'analyse du contrôle de l'écoulement, puis les différents modes de régulation de l'écoulement fluvial. L'équation d'emmagasinement ou équation du bilan hydrique (relation mathématique entre la capacité d'accumulation du réservoir, les sorties d'eau et la sécurité de l'exploitation en eau) est expliquée en insistant sur l'importance des critères de sécurité de l'alimentation en eau et du volume escompté des prélèvements. Pour résoudre l'équation d'emmagasinement on peut avoir recours soit à des méthodes déterministes, soit à des méthodes stochastiques. La note explique les méthodes déterministes (techniques numériques et graphiques), les méthodes stochastiques (techniques de simulation, techniques analytiques et numériques) et les méthodes semi-stochastiques de résolution de l'équation d'emmagasinement. En conclusion, le chapitre expose les solutions précalculées de l'équation d'emmagasinement et l'influence qu'a la période d'échéance sur la régulation en régime d'été. Les méthodes d'analyse des crues et les problèmes hydrologiques posés par les crues sont exposés au chapitre 4. Celui-ci donne d'abord une définition, puis analyse les caractéristiques de la crue (niveau, débit, volume, durée, manifestation saisonnière et vitesse du courant) avant d'exposer les objectifs de l'analyse des crues qui consiste essentiellement à évaluer dans quelle mesure et à quelle fréquence les crues perturbent la vie et les activités normales d'une population. Il explique comment définir la fréquence des crues (méthodes fondées sur les séries des pointes de crues maximales annuelles et sur les séries de valeurs supérieures à une valeur donnée) et analyse les méthodes et les problèmes relatifs à l'établissement de modèles et à l'estimation de la distribution de fréquence des crues. L'auteur expose en détail les techniques et méthodes de synthèse des crues qui comprennent notamment la détermination indirecte des écouléments des crues en utilisant la formule rationnelle ainsi que d'autres formules empiriques régionales, les méthodes de synthèse des courbes de débit de crues (méthodes géométriques, méthodes de corrélation, méthodes d'égale probabilité et la méthode de la crue maximale probable), les calculs de progression des crues pour déterminer approximativement l'écoulement en régime non permanent dans les cours d'eau et à travers les réservoirs d'accumulation (calcul de la propagation des crues en absence et en présence d'un réservoir) et les méthodes de protection contre les inondations comportant l'utilisation de réservoirs d'accumulation avec débouchés réglables ou non, le calcul d'un réservoir pour la maîtrise des crues et enfin la protection contre les inondations au moyen d'un réservoir exploité à des fins multiples.
РЕЗЮМЕ

В настоящем отчете излагаются различные методы и концепции, связанные с практическими применениями гидрологии и эффективным использованием гидрологических методов при планировании, проектировании и руководстве проектами в области ресурсов поверхностных вод.

В главе 1 приведено общее описание водохозяйственной деятельности. Описываются новые тенденции, концептуальные изменения и постепенное расширение границ этой дисциплины, включая принятие новых методов анализа (анализ систем, статистические и вероятностные концепции) и широкое применение быстroredействующих цифровых ЭВМ.

Одним из применений гидрологии в водохозяйственной деятельности является количественная оценка имеющихся водных ресурсов в так называемом водном кадастре. В главе 2 описаны гидрологические проблемы, связанные с характеристиками поверхностного стока. Приводятся методы получения максимального количества информации по данным регистрации, полученным с единичных станций и методы оценки их надежности. Описываются характеристики ежегодного стока, включая средние значения, коэффициент изменчивости, вероятностное распределение (свойства распределений стока, модели распределения ежегодного стока, подготовка моделей распределения и графическое изображение функций распределения), характеристики временных флуктуаций и методы моделирования последовательности ежегодных стоков (моделирование произвольной последовательности и цепь Маркова первого порядка). Приводятся флуктуации стока за часть года, описанные с помощью средних ежемесячных и средних ежесуточных стоков, а также объясняются соответствующие методы анализа для моделирования серий ежемесячных стоков, включающие использование кривой продолжительности для средних ежесуточных стоков. Подробно описывается применение статистических методов (формулы и модели регрессии) и детерминистских моделей (методы с использованием соотношений и распределенных параметрами) для обобщения зарегистрированных данных стока с единичной станции, а для статистического подхода рассматриваются влияние корреляции на эффективность обобщения зарегистрированных данных и распространенных ошибок регрессии. Косвенное определение стока в неизмеренных бассейнов может проводиться с использованием аналогичных гидрологических методов (концепция расчетного и аналогового бассейна), а также на основе региональных характеристик стока, которые включают оценку конкретного стока, коэффициент стока, региональный коэффициент изменчивости ежегодного стока и оценку вероятностного распределения комбинированного ежегодного или сезонного стоков двух или более рек в данном районе.
Осенняя роль гидрологии в водохозяйственной деятельности связана с процессом соответственно имеющимся ресурсов с потребностями в них и включает проблемы, связанные с недостатком воды, с одной стороны, и их избытком (наводнений), с другой. В главе 3 описываются гидрологические проблемы, связанные с недостатком воды. Рассматриваются различные интерпретации засух и описываются цели анализа (продолжительность засух, вероятность возникновения, интенсивность, время возникновения и распространение по площади) и методы анализа (эмпирические методы, методы Монте-Карло и аналитические методы). Термин "недостаток воды" объясняется с точки зрения потребностей в воде, спроса и предложения, использования воды, потерь воды и водного дефицита. Приведено подробное описание регулирования стока с помощью водохранилищ, включая определения и характеристики, элементы, учитываемые при анализе регулирования стока и их характеристики, и типы регулирования стока. Приводится уравнение неизвестности (математическая зависимость между запасом воды в водохранилище, пополнением воды из водохранилища и недостатком) с описанием характеристик недостатка и целевого значения попуска из водохранилища. Методы решения уравнения неизвестности могут быть либо детерминистскими, либо стохастическими. Описываются детерминистские методы (численные и графические методы), стохастические методы (моделирование, аналитические и численные методы) с полустохастическими методами решения уравнения неизвестности. Рассматриваются предварительно рассчитанные решения уравнения неизвестности и влияние временного горизонта на регулирование меженного стока.

В главе 4 описаны методы анализа паводков и гидрологические проблемы, вызываемые паводками. Приводится аналитическое исследование, включающее определение паводка, его характеристики (подъем воды, расход, объем, продолжительность, сезонное возникновение и скорость течения воды) и цели анализа паводка, включая оценку степени и частоты вмешательства паводков в обычную жизнь и деятельность населения. Описываются методы определения частоты паводков (методы, основанные на ежегодных максимумах и сериях частичной продолжительности) и методы и проблемы, связанные с моделированием и оценкой реакции частоты паводков. Подробно описываются методы и техника синтеза паводка, которые включают носовное определение паводковых стоков с использованием рациональной формулы и других региональных эмпирических формул, методы синтеза гидрографа паводка (геометрические методы, методы корреляции, методы равной вероятности и метод максимального вероятного паводка), методы расчета гидрографа стока (с учетом трансформации паводка) для приближенного решения уравнения неустойчивого потока воды в русле реки и через водохранилище (направление паводковой волны в водохранилище и русле) методы регулирования паводков, включая регулируемое и нерегулируемое хранение воды в водохранилище, проектирование водохранилищ для регулирования паводков и регулирование паводков с помощью многоцелевого водохранилища.
RESUMEN

En este informe se exponen los distintos métodos y conceptos que intervienen en las aplicaciones prácticas de la hidrología y se describe la manera de utilizar con eficacia las técnicas hidrológicas en la planificación y realización de los proyectos que tratan de las aguas superficiales.

En el Capítulo 1 se estudia de manera general la ordenación de los recursos hidrícos y se especifican las nuevas tendencias y conceptos así como el gradual desarrollo de esta disciplina, que implica la adopción de nuevos métodos de análisis (análisis de sistemas, conceptos estadísticos y de probabilidad) y la utilización cada vez más amplia de las computadoras numéricas de gran velocidad.

En las aplicaciones de la hidrología a los recursos hidrícos se incluye la evaluación cuantitativa del agua disponible, dentro del denominado balance de dichos recursos. En el Capítulo 2 se exponen los problemas que se plantean para determinar el balance de la escorrentía superficial. Se explican también las técnicas que han de utilizarse para extraer la mayor cantidad de información de los registros de una sola estación y los métodos para valorar su seguridad. También se discuten las características de la escorrentía anual, incluida la escorrentía media, el coeficiente de variación, la distribución de probabilidad (propiedades de las distribuciones de escorrentía, modelos de distribución para la escorrentía anual, ajuste de los modelos de distribución y expresión gráfica de las funciones de distribución), la estructura de las fluctuaciones en el tiempo y los métodos de simulación de series de la escorrentía anual (simulación de una serie aleatoria y de la cadena de Markov de primer orden). También se estudian las fluctuaciones subanuales de la escorrentía, descritas mediante los flujos medios mensuales y medios diarios, y se explican los correspondientes métodos de análisis para la simulación de la serie de flujo mensual, incluyendo la utilización de la curva de duración para los flujos medios diarios. Se estudia ampliamente el uso de métodos estadísticos (fórmulas y modelos de regresión) y de los modelos determinísticos (modelos de elementos concentrados y modelos de distribución) para ampliar los registros del caudal de la corriente de una sola estación. Por lo que respecta al planteamiento estadístico se estudia el efecto que ejerce la correlación sobre la eficacia de la ampliación del registro, así como también los errores de las regresiones. Se puede hacer una determinación indirecta del caudal de una corriente en las cuencas en donde no existan estaciones fluviométricas, utilizando la técnica de analogía hidrológica (fundada en el concepto de cuencas experimentales y cuencas análogas) y también mediante la utilización de las características regionales de la escorrentía en donde se incluye la estimación de la escorrentía específica, del coeficiente de escorrentía, del coeficiente regional de la variación de la escorrentía anual y la estimación de la distribución de probabilidad de la combinación de escorrentía anual o estacional de dos o varios ríos de determinada región.

La función básica de la hidrología en la ordenación de recursos hidrícos es ajustar los recursos de agua disponibles a las necesidades y lleva consigo la resolución de determinados problemas que se plantean con motivo de la deficiencia de agua por una parte y del exceso de la misma (inundaciones) por otra. Los problemas
hidrológicos que son consecuencia de la deficiencia de agua se estudian en el Capítulo 3. Se estudian las diferentes interpretaciones de las sequías, la finalidad de los análisis (duración de la sequía, probabilidad de ocurrencia, intensidad, tiempo de ocurrencia y extensión superficial) así como los métodos de análisis (métodos empíricos, métodos de Monte Carlo y métodos analíticos). Se explica la significación del término "deficiencia de agua" en lo que respecta al agua que se necesita, a las demandas de agua, su utilización, y a las pérdidas y déficit de la misma. También se trata con todo detalle de la regulación de las corrientes mediante embalses y se definen las características de los mismos, así como los elementos que hay que considerar para analizar el control de las corrientes de agua, sus características y tipos de regulación de las corrientes. Se describe la ecuación de almacenamiento (relación matemática entre el agua almacenada en un embalse, el agua salida del embalse y la seguridad) haciendo una exposición de las características de la seguridad y de la cantidad de agua que ha de salir de un embalse para satisfacer todas las necesidades. Los métodos utilizados para resolver la ecuación de almacenamiento pueden ser determinísticos o estocásticos. Se discuten los métodos determinísticos (técnicas numéricas y gráficas), los métodos estocásticos (simulación, técnicas analíticas y numéricas) así como los métodos semiestocásticos para resolver la ecuación de almacenamiento. También se estudian las soluciones precalculadas de la ecuación de almacenamiento y el efecto del factor tiempo en la regulación del estiaje.

En el Capítulo 4 se describen las técnicas de análisis de las inundaciones y los problemas hidrológicos por ellas causados. Se expone un estudio analítico en donde intervienen la definición de inundación, sus características (altura de la inundación, caudal, volumen, duración, ocurrencia estacional y velocidad de la corriente) así como los objetivos del análisis de las inundaciones, que implica la evaluación del grado y frecuencia de interferencia de las inundaciones con la vida normal y actividades de la población. Se estudian también los métodos para definir la frecuencia de las inundaciones (métodos fundados en las series de valores máximos anuales y en las de duración parcial) así como los problemas que se plantean para reflejar en un modelo y estimar la distribución de la frecuencia de las inundaciones. Se hace una exposición detallada de las técnicas y métodos de síntesis de las inundaciones en los cuales se incluye la determinación indirecta de los flujos de la inundación utilizando la fórmula racional y otras de carácter regional y empírico, los métodos de síntesis del hidrograma de las inundaciones (métodos geométricos, de correlación, métodos de igual probabilidad y método de la inundación máxima probable). Se trata también de las técnicas de desviación de las corrientes para resolver el problema del flujo irregular en los cauces de los ríos utilizando también los embalses (desviación de la corriente atravesando el embalse o de otros cañales). Se exponen también las técnicas de control de las inundaciones utilizando el almacenamiento controlado o no en los embalses, las técnicas de diseño de un embalse de control de inundaciones y la utilización para este fin de embalses polivalentes.
CHAPTER 1

WATER RESOURCES MANAGEMENT

During the last two decades, water resources management has established itself as a relatively self-contained field concerned with seeking optimal general solutions to problems arising from the demand of society for water on the one hand and the availability of water in nature on the other hand.

In seeking the proper balance between these two components, it is essential to view them as elements inseparable from the physical and social environment. Although the two facets of water (availability and demand) represent the core in the absence of which water resources management has no meaning, it will usually not be they themselves but political, ecological and other considerations which will play the most important role in the decision-making process concerned with the management of water resources.

To put the applications of hydrology to water resources management into a proper perspective, Figure 1 (reproduced from (1)) gives a general outline showing the place of these two disciplines in a broader context as well as their relation to each other.

The broad concept of water resources management outlined in Figure 1 has evolved only during the most recent years. It reflects the changing attitudes of man toward himself and the gradually growing awareness that, natural resources being limited as they are, their more responsible and well-co-ordinated management is one of the main prerequisites for sound development of human society.

1.1 HYDROLOGY AND WATER RESOURCES MANAGEMENT

Because of the close relationship between hydrology and water resources, the state of, and changes in, one discipline always have had a profound impact on the other. To appreciate the dynamics inherent to this relationship one has to realize that the source of the driving force has always been the field of water resources management. Even before this field had been recognized as such, its latent nucleus, the interface between water and society, always existed and functioned as the principal stimulus for the development and scientific progress of hydrology.

Before discussing the applications of hydrology to water resources management it is therefore necessary to outline briefly the new trends in contemporary water resources management. It is the general philosophy behind this report to present mainly those hydrological methods and to outline those problems which respond to the needs of modern water resources management.

1.2 NEW TRENDS IN WATER RESOURCES MANAGEMENT

Perhaps the most important new trend in water resources management has been the gradual broadening of its scope. Not so long ago, water resources management
consisted of ad hoc solutions of the "water demand-supply" problem as it emerged in connexion with some particular engineering project like an irrigation or water power scheme. The problem was considered to be one of a purely engineering nature, similar to those of concrete composition, foundation arrangement, structure stability, etc. Only with the increasing sizes and growing numbers of projects has it become evident that all the individual "optimal" solutions to particular water management problems add up to a chaotic conglomerate of conflicting situations whose compound effect on the economy is far from optimal and in many cases downright detrimental.

This experience has been responsible for the relatively new trend shifting water resources management from the level of engineering technology to that of environmental planning. Such trends are reflected by the growing tendency at universities to identify the discipline of water resources management, and consequently that of hydrology as well, with divisions of environmental or Earth sciences as opposed to the classical arrangement where they are part of the area of civil engineering. Similar trends can be observed in some countries in their administration of water resources at governmental level.*

The main consequence of the broad concept of water resources management has been an enormously increased complexity of the latter which necessitated the adoption of new techniques and methods of analysis. Systems analysis (operations research) backed by extensive use of high-speed computers has been the answer to this challenge.** The systems approach to water resources management has introduced into this field modern mathematical concepts such as decision theory, theory of games, mathematical modelling, control theory, mathematical statistics, probability theory and simulation techniques.

A very recent new feature of water resources management is its increasing sensitivity to the element of time. This has to do with the exponentially accelerating rate of population growth and related problems like disastrous rate of increase of environmental pollution, depletion of natural resources, and technological changes. All this places high requirements on the flexibility of water resources management, shifts the emphasis from static to dynamic concepts, and increases the importance of evaluating the uncertainty inherent in present solutions, decisions and projections.

* Canada, for example, established a Department (Ministry) of the Environment in 1971, where the Inland Waters Branch of the Department of Energy, Mines and Resources has been transferred.

** The Harvard Water Program as documented in the now already classical book by A. M. Maass et al., Design of Water-Resources Systems, Harvard University Press, Cambridge, Mass., 1963, can be considered the turning point in water resources management.
1.3 NEW TRENDS IN HYDROLOGY

Hydrology has been affected not so much by conceptual changes in water resources management, but rather by the methodological apparatus newly adopted in this discipline, by new requirements on hydrological inputs into the water resources management process, and finally by new techniques of quantitative analysis.

Systems approach has found a very fertile ground in hydrology and the crop of system-oriented mathematical models of basins is growing rapidly. Two basic trends can be identified, namely the lumped-system and the distributed-system approach. The first treats a basin as one element (e.g. the unit hydrograph concept), the second divides it into several geographical areas which are more or less homogeneous from a certain point of view (e.g. elevation zones, zones of different terrain slopes, zones of different vegetation cover, etc.) and synthesizes the desired basin output (e.g. runoff, infiltration, etc.) by properly combining the outputs from individual elements. Basically, the systems approach is not concerned with the physical mechanisms transforming inputs into outputs but specifies the given element, the "black box" (geographical area, basin), in terms of its input and output properties.

The growing importance of the element of risk, resulting both from the necessity for a quantitative assessment of reliability of the forecasts and projections entering into the water-management decision-making process, and from the increasing awareness of the complexity of hydrological processes and of their inherent uncertainties, has intensified the use in hydrology of a broad spectrum of statistical and probabilistic concepts like hypothesis testing, distribution theory, multivariate analysis, and stochastic processes.

Technical requirements of sophisticated analyses and the ever-growing amounts of data to be processed have led to extensive use of high-speed digital computers. On the other hand, the widespread use of computers has itself a profound influence on hydrological analysis by promoting the application of numerical methods, digital simulation and Monte Carlo techniques. A rather unfortunate side effect of the computerization of hydrology has been an increased alienation between the hydrologist and the data he is working with. It is often the case that there is less insight into both data and results of analysis, and less opportunity to detect errors, to interpret anomalies correctly, and to acquire a "feeling" for the phenomenon analysed.

1.4 HYDROLOGICAL ASPECTS OF WATER RESOURCES MANAGEMENT

The role of hydrology in water resources management is to provide data (in a broad sense) regarding the time and space distribution of water on land. Thus water resources management is in fact not interested in hydrology as such, that is to say in the dynamics of hydrological processes, in how things happen and why, but rather in the quantitative results that hydrology can offer.

The first sphere of interest is the quantitative assessment of available water resources which, combined with their qualitative assessment, represents the so-called water resources inventory. The latter is one of the two fundamental components of water resources management; the other is water needs and lies outside the scope of hydrology (Figure 1).
Figure 1 - Scheme showing the place of water resources management in the decision-making process
The second sphere of interest of water resources management in hydrology relates to the process of matching the available supplies with the needs and involves problems arising from water deficiency on the one hand, and those arising from water excess on the other hand. Here the role of hydrology is twofold. Firstly, to evaluate quantitatively the natural régime of water extremes which is in fact a rather specialized extension of its above-stated general role; secondly, to evaluate quantitatively the effect of the contemplated artificial alterations of the natural water régime. From the water resources management point of view, the purpose of the first type of analyses is to assess the danger of the water-régime extremes, while the purpose of the second type is to assess the efficiency of various solutions reducing that danger.

The above outline provides a natural overall framework of this report which will be as follows:

Hydrological problems in water resources inventory;
Hydrological problems arising from water deficiency;
Hydrological problems arising from water excess.
For water resources management, the basic task of hydrology is to supply data on the time and space distributions of that part of the existing surface and groundwater which is directly manageable, that is, which can be redistributed in time and space by conventional means like dams, canals, pipes, pumps, and perhaps other engineering structures and machines.

Three major sources of water come into this category, in particular streamflow, water in lakes, and water in underground storage. However, in this report we shall deal only with streamflow; the other two water sources lie outside its terms of reference, being the subjects of physical limnology and groundwater hydrology, respectively.

The terms streamflow and runoff (meaning surface runoff) will be often used synonymously in this report. Although their original meaning is distinct (streamflow has a connotation of an "instant" rate of flow while runoff is the total amount of water that has passed through a given stream cross-section within some longer period of time, e.g. a year), there are many instances where a clear distinction is irrelevant and both terms convey more or less the same idea, for instance, "streamflow regulation" and "runoff regulation", or "annual runoff" and "mean annual streamflow" if used merely as a quantitative characteristic of the overall wetness or dryness of a year. As seen from the second example, the term runoff can sometimes simplify the terminology.

Generally three levels of streamflow data are utilized in water resources management to assess the quantity of water available in a stream, in particular the mean annual runoff, the mean monthly flows, and the mean daily flows. Average flows for shorter intervals than a day are seldom employed in water resources planning and design, except for flood control and, sometimes, hydro-power generation.

As far as the quality of the data is concerned, there are recommended limits for the accuracy in measuring individual variables like flow velocity, cross-sectional area of a stream, etc., but there are no standards relating the overall quality of runoff data (their accuracy and representativeness) to particular types of water resources management tasks or projects. It is recognized that such standards would be rather academic since the demand for various water resources management projects is a highly dynamic social factor whose fulfilment can seldom be subordinated to however well-intentioned hydrological standards. Under such circumstances the recommended philosophy is to extract from available data the most information possible and to project the actual degree of hydrological uncertainty into the water management decisions.
2.1 INFORMATION OBTAINABLE FROM A SINGLE STATION

Records of a stream-gauging station represent the primary source of information on streamflow. As will be shown in section 2.2, it does not happen often that this basic information can be significantly improved by using other data like streamflow records from other stations or meteorological records. Direct streamflow records still are, and for a long time to come will be, the most reliable and desirable source of information on streamflow. It is therefore imperative to extract the maximum amount of information from these records and to know how to assess its reliability.

2.1.1 Annual runoff

The overall quantity of surface water that is available in a basin for water resources management is well characterized by runoff volumes of individual years or by the corresponding mean annual flows. The latter variable is usually used for descriptive purposes since, being expressed in m$^3$ s$^{-1}$ (or ft$^3$ s$^{-1}$), it offers the desirable possibility of using numbers of lower order than is the case with annual runoff volume. However, in water supply studies, reservoir design, irrigation projects, and other instances involving water quantities rather than flow rates (for instance in economic studies where a unit volume of water is assigned monetary value), the annual runoff volume is preferable.

A sequence of mean annual flows contains a considerable amount of information indispensable for quantitative water management but not all of it can be easily extracted from the data, especially if the recorded runoff series is short. However, the cream of the information is contained in two parameters, the mean and the variance (or, derived from it, the coefficient of variation) characterizing, respectively, central tendency and variability of annual runoff. Estimates of these two parameters are easily obtained from empirical data as well as estimates of their errors. In fact, it is often the case that these two parameters are the only ones that can be estimated with a reasonable accuracy and thus relied on in water management decisions. All other parameters and properties of annual runoff, although important for assessing the water resource concerned, are generally much less reliable because of their dependence on assumptions whose validity can hardly be verified on the basis of small data samples typical for annual runoff records.

2.1.1.1 Mean annual runoff

The mean annual runoff (the average of annual runoffs) characterizes the maximum potential quantity of water that is available from a basin in the long run. As such it is of fundamental importance for a surface water inventory where it is used as a basic parameter, usually in the form of the long-term mean flow. Besides serving the general purpose of defining the upper limit of available surface water supply, it is used as a unit in terms of which annual runoff of current years is expressed (hence also termed the norm of runoff) as well as, for instance, amount of runoff in individual seasons (snowmelt runoff). In storage reservoir design, its
value relative to the dam site is often used as a unit for measuring the reservoir storage capacity. This makes it possible to get a rough idea about the regulating capability of reservoirs at different locations along one river or on different rivers.

The value of mean annual runoff obtained from streamflow records is not its true value but only a sample mean, and it is important to know how accurately such a sample mean computed from \( n \) years of records, \( \bar{Q}_n \), represents the true mean annual runoff \( Q \). This accuracy is measured by the standard error of \( \bar{Q}_n \) which, for practical purposes, is best expressed in per cent of \( \bar{Q}_n \) and given by the formula

\[
S(\bar{Q}_n) = \frac{C_v}{\sqrt{n}} \times 100 \% 
\]

(1)

where \( C_v \) is the estimated coefficient of variation of annual runoff (paragraph 2.1.1.2).

Standard errors of mean annual runoff for typical values of \( C_v \) are given in Table I.

### TABLE I

Standard errors of the mean (in per cent of the latter), computed using equation (1)

<table>
<thead>
<tr>
<th>Coefficient of variation ( C_v )</th>
<th>Sample size, ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 10 20 50 100</td>
<td></td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>0.10</td>
<td>4.5 3.2 2.2 1.4 1.0</td>
</tr>
<tr>
<td>0.20</td>
<td>8.9 6.3 4.5 2.8 2.0</td>
</tr>
<tr>
<td>0.40</td>
<td>17.9 12.7 8.9 5.7 4.0</td>
</tr>
<tr>
<td>0.60</td>
<td>26.8 19.0 13.4 8.5 6.0</td>
</tr>
<tr>
<td>0.80</td>
<td>35.8 25.3 17.9 11.3 8.0</td>
</tr>
<tr>
<td>1.00</td>
<td>44.7 31.6 22.4 14.1 10.0</td>
</tr>
</tbody>
</table>
The testing of accuracy of the mean is done by means of the so-called t distribution (the procedure as well as tabulated values of t are given in elementary statistical literature). For a rough estimate one can assume that the true mean $\bar{Q}$ will be within the limits

$$\bar{Q}_n \pm 2 S(\bar{Q}_n) \text{ with 95 per cent reliability and}$$

$$\bar{Q}_n \pm 3 S(\bar{Q}_n) \text{ with 99.7 per cent reliability.}$$

Thus, for example, if the mean flow of a river having $C_v = 0.20$ (corresponds to humid region) is found to be $100 \text{ m}^3 \text{ s}^{-1}$ on the basis of a 20-year streamflow record, there is about a 95 per cent chance that the true long-term mean $\bar{Q}$ will be within 91 and $109 \text{ m}^3 \text{ s}^{-1}$, and about a 99.7 per cent chance that it will be within 86.5 and $113.5 \text{ m}^3 \text{ s}^{-1}$.

It should be pointed out that, given streamflow record of a specified length, the mean annual runoff is the characteristic that can be ascertained with the highest accuracy. This is the same as saying that if working with a specified accuracy level, the determination of the mean annual runoff has the lowest requirement on the length of streamflow record.

2.1.1.2 Variability of annual runoff

The value of mean annual runoff does not give any idea as to how much the runoff in individual years can differ from the average. However, such deviations are of considerable practical importance and represent, in fact, one of the main sources of concern in water resources management. As a rule, the greater the variability of annual runoff the more difficult and expensive becomes the management of water quantity. For example, a reservoir designed to be able to maintain its rate of outflow at, say, 80 per cent of the long-term mean flow may require as much as three times larger storage capacity on a river with runoff variability characterized by $C_v = 0.5$ than on a river with equal mean runoff but with $C_v = 0.3$.

The basic measure of variability is the square root of the variance called the standard deviation, $\sigma$. Its value for annual runoff, based on $n$ years of records, is given as

$$\sigma_n(Q) = \sqrt{\frac{\Sigma (Q - \bar{Q}_n)^2}{n-1}}$$

However, to enable comparisons of runoff variabilities for points with different values of annual runoff, the dimensionless coefficient of variation, $C_v = \sigma_n(Q)/\bar{Q}_n$, is preferable.

Again, it is important to know the accuracy with which $C_v$ can be obtained from an $n$-year streamflow record. Although the standard error, $S(C_v)$, depends on the
distribution type of the variable (annual runoff in this case) which is not easy to find (see paragraph 2.1.1.3), the differences for different types are fortunately not too large. Equations giving $S(C_v)$ for normally and gamma distributed variables can be found for instance in Sokolovskiy, 1959 (2). Table II gives $S(C_v)$ in per cent of $C_v$ for normally and gamma distributed variables.

Comparing Tables I and II, it is evident that the standard error of the coefficient of variation of annual runoff is much larger than error of the mean annual runoff, especially for rivers in humid-to-temperate climatic regions where runoff variability tends to be low. So for the example given in the preceding paragraph, the standard error in $C_v(Q)$ would be about four times as large (percentage wise) as that in $Q$. However, in most practical cases $S(C_v)$ does not exceed 25 per cent, which is still acceptable compared to errors in other parameters.

**TABLE II**

Standard errors of the coefficient of variation (in per cent of the latter), for gamma and (in brackets) Gaussian distribution

<table>
<thead>
<tr>
<th>Coefficient of variation $C_v$</th>
<th>Sample size, n</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>0.10</td>
<td>36 (32)</td>
</tr>
<tr>
<td>0.20</td>
<td>37 (33)</td>
</tr>
<tr>
<td>0.40</td>
<td>42 (37)</td>
</tr>
<tr>
<td>0.60</td>
<td>50 (42)</td>
</tr>
<tr>
<td>0.80</td>
<td>60 (48)</td>
</tr>
<tr>
<td>1.00</td>
<td>70 (55)</td>
</tr>
</tbody>
</table>
In testing the accuracy of \( C \), one has to test the variance \( \sigma^2(Q) \) by using the \( \chi^2 \) distribution (procedures and relevant tables given in elementary statistical literature), and then work out the corresponding limits for \( C \) from the relation

\[
C = \sigma \sqrt{Q}.
\]

### 2.1.1.3 Distribution of annual runoff

The mean and the coefficient of variation alone are seldom sufficient for purposes other than a general assessment of runoff conditions in a basin or river, and for basic comparisons of these conditions among different basins or rivers. Their information content is too low for providing a reliable basis for answering such questions as what is the chance of the annual runoff dropping below, say, 50 per cent of the mean, or the chance of exceeding a certain value, or of the likelihood of several dry or wet years occurring in a row.

For answering such questions, which are typical for water management, the necessary (but not always sufficient) condition is to know the probability distribution of annual runoff. With this problem, applied hydrology enters rather shaky ground, mainly because of the shortness of the available streamflow records.

**2.1.1.3.1 Properties of runoff distributions**

Three important properties of probability distribution of runoff can be derived from the physical nature of runoff: first, its shape is asymmetrical; second, the distribution is limited in one direction (lower tail) and unlimited in the other (upper tail); third, its absolute lower limit is zero since runoff cannot assume negative values.

Strictly speaking, there obviously is some theoretical upper limit to the amount of annual runoff, at least in the sense of the finite quantity of water on our planet. However, this is certainly irrelevant for any water-management purpose since for any stream-gauging station the probability of annual amount of runoff approaches asymptotically zero for values incomparably lower than such absolute limit. From a practical point of view it is therefore reasonable to regard the runoff distribution as unlimited in the upper direction. It then automatically follows that, being limited in one direction and unlimited in the other, the runoff distribution must be asymmetrical; incidentally, this conclusion is valid for any runoff distribution, not only for that of annual runoff.

Asymmetry being an intrinsic property of runoff distribution, a measure of asymmetry is an important characteristic of distribution shape.

A simple characteristic of asymmetry is the distance from the mean to the mode, but the most generally employed parameter is the coefficient of asymmetry, or coefficient of skewness, \( C \). Its estimate from an \( n \)-year series of annual runoff is given by
HYDROLOGICAL PROBLEMS IN WATER RESOURCES INVENTORY

\[ C_s(Q) = \frac{n}{(n-1)(n-2)} \frac{\sum(Q_i - \bar{Q})^3}{\sigma_n^3} \]  

(3)

Standard error of \( C_s(Q) \) strongly depends on the distribution of the original variable \( Q \), so that without knowing the distribution of annual runoff in advance one does not know how accurate the estimate of \( C_s \) is. On the other hand, without knowing the limits within which \( C_s \) can be expected, one can hardly judge the type of distribution of \( Q \).

By and large, it is very difficult to make reliable inferences about the asymmetry of annual runoff. It may well happen that the computed \( C_s(Q) \) is positive but its true value can be negative or vice versa.

As an example of the uncertainty involved, Table III lists standard errors of \( C_s \) for the case that annual runoff has a gamma distribution where the correct value of \( C_s \) is equal to \( 2C_v \), and a normal distribution (formulae for standard error of \( C_s \) can be found in (2)).

Taking the previously used example and assuming that annual runoff in that case has a gamma distribution and for that reason the correct \( C_s(Q) = 0.4 \), one finds that standard error of its estimate is 153 per cent. Thus assessing the accuracy of the computed value of \( C_s \) on, say, a 95 per cent reliability level (and assuming that \( C_s \) itself is approximately normally distributed), the limits within which the true value can fluctuate are found to be about 300 per cent from the value obtained using equation (3). The only conclusion one can make on this basis, is that there is no reasonably reliable way to estimate the true value of \( C_s \) of annual runoff in the case shown and, for that matter, no way to verify the original assumption of \( Q \) having a gamma distribution.

2.1.1.3.2 Distribution models for annual runoff

Given the difficulties with the estimation of the true value of the coefficient of skewness, use has been made of the fact that annual runoff of most rivers seems to exhibit positive skewness. Positive skewness has become an a priori assumption for annual runoff distributions and positively skewed models have gained considerable popularity, among them mainly the gamma and the log-normal types.

They both have one appealing feature: they do not require the coefficient of skewness to be calculated from the data and can be fitted just on the basis of the mean and the coefficient of variation (they both are two-parameter distributions).
TABLE III

Standard errors of coefficient of skewness (numbers in brackets give standard errors in per cent of $C_s$)

<table>
<thead>
<tr>
<th>Coefficient of variation</th>
<th>Sample size, n</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>$C_v$</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
</tr>
</tbody>
</table>

Gaussian distribution

$C_s = 0$

Any value of $C_v$ | 1.10 | 0.78 | 0.55 | 0.35 | 0.24

The coefficient of skewness is a function of $C_v$, namely

- $C_s = 2C_v$ for gamma distribution, and
- $C_s = 3C_v + C_v^3$ for the log-normal type.

However, by using these models, one does not eliminate the intrinsic uncertainty about the runoff distribution. The uncertainty in estimating $C_s$ from the data is merely replaced by that of arbitrarily choosing it regardless of data. This may be even worse since there is no evidence that the above two relations are typical...
for annual runoff. As shown by Klemeš (3), it is reasonable to expect that many runoff series possess much smaller values of $C_s$ and that $C_s$ may even be negative, especially for rivers with large storage capacities in their basins, or for those fed by snow and glaciers.

Allowing for the flexibility in $C_s$, Kritskiy and Menkel (4) recommended, as early as 1946, a power transformation of the gamma distribution to be used for annual runoff. In this case a variable $aQ^b$ (where $a$ and $b$ are parameters that can be expressed in terms of $C_s$), rather than $Q$, is assumed to be gamma distributed. This distribution is widely used in the U.S.S.R. and its distribution functions are tabulated in most Russian books on hydrology (see, e.g., (2)).

Flexibility in skewness can also be achieved by including a so-called location parameter $c$ into the gamma or log-normal distribution. In so doing, it is assumed that a variable $Q-c$ rather than $Q$ possesses the given distribution. Although satisfactory for many practical purposes, this procedure raises one objection in that it postulates the absolute lower limit of annual runoff to be equal to $c$ rather than to zero, which is hardly justifiable. The gamma distribution with a location parameter is commonly known as a Pearson type III model; in Russian hydrological literature it is often referred to as a binomial distribution.

In 1965 Markovic (5) used the gamma and the log-normal distributions, both the basic ones and those with location parameters, and also the Gaussian distribution, to fit 446 samples of annual runoff (the average sample size was 37 years). He found that not one of the five types used could be rejected as unacceptable for most of the samples if current statistical tests were used.

2.1.1.3.3 Fitting of distribution models

Freehand fitting consists of plotting the data arranged in order of magnitude on probability paper at proper plotting positions (see later) and drawing a line by eye through the plotted points. This method is highly subjective and cannot be recommended for extrapolation of values of extreme probabilities. Moreover, the fitted line cannot be easily described for it does not represent any specified distribution type. However, the data can be plotted on probability paper with a special abscissa scale where a certain type of distribution function appears as a straight line which is then fitted to the plotted points; this technique gives a freehand fit of a specified distribution type. The following types of probability paper are generally available (distribution type whose distribution function appears as a straight line is given in brackets): linear (Gaussian), logarithmic (log normal), double logarithmic (Goodrich), Brovkovich (gamma), Gumbel (extremal type I), Fréchet (log-extremal). Samples of these probability papers can be found, for instance, in reference (6).

The most simple but least accurate objective method of fitting is the method of moments. In this case, the estimates of the mean and the coefficient of variation are obtained from the data and regarded as the true parameters of the distribution model chosen. Any two-parameter distribution can be fitted in this manner. By using the estimate of $C_s$ (equation (3)), one can use this method for three-parameter distributions (e.g. the power transformation of the gamma type, Pearson III, etc.).
The method of maximum likelihood is recommended as the most reliable but it usually is also the least convenient one and is seldom used in routine engineering applications (it was used, for instance, by Markovic in (5)).

A method which can be recommended wherever a digital computer is available, is that of least squares (it can be applied even with the aid of a mechanical desk-calculator without much inconvenience). It consists, in principle, in the following:

First, the empirical distribution function of the data is set up by arranging the data in order of magnitude and assigning to each member $Q_i$, $i = 1, 2, \ldots, n$, of the ordered array a cumulative-probability co-ordinate called plotting position. The latter can be calculated by using some approximate formulae, for instance

$$p = \frac{m-0.3}{n+0.4} \quad \text{or} \quad p = \frac{m}{n+1},$$

where $n$ is the sample size (number of years), and $m$ is the order number of individual member. Then a distribution model is chosen and the fit obtained by the method of moments is used as the first approximation. From the latter the first estimate of the ordinates, $Q'_1$, is obtained and the sum

$$\sum_{i=1}^{n} (Q_i - Q'_i)^2$$

calculated. In the next step, the value of the least reliable parameter ($C_v$ when a two-parameter distribution is used, $C_s$ in case of a three-parameter one) is slightly changed in the model and new values of all the $Q'_i$ and of the above sum are calculated. If the new sum is smaller than the original one, the least reliable parameter is further changed in the same direction and the procedure is repeated until the sum of squares reaches a minimum. If the second sum is greater than the original one, the changes in the parameter are made in the opposite direction until a minimum is reached. The value of the changing parameter, corresponding to a minimum value of the sum of squares, is then considered to be the best fit. Standard programmes of least-square procedures are available with most computers.

The quantile method devised by Alekseyev (1962) is a shortcut technique formalizing the freehand fitting. It consists in the following: a curve is fitted by eye to the descending sequence of points plotted on probability paper, values of the variable $Q'$ are read from it for probabilities 5, 50 and 95 per cent and used for computing an auxiliary variable $S$. Then using a table giving a precomputed relationship between $S$ and distribution parameters for various distribution types, one finds the value of $C_s$ and $C_v$. The distribution function based on these parameters is regarded as the best fit.

Examples of using this method for fitting the Pearson III and log-normal distributions, together with relevant tables, can be found in WMO Technical Note No. 98, Estimation of maximum floods (7).

Another similar method by Jenkinson, called the method of sextiles, is also described in (7). In Jenkinson's method the empirical distribution function is divided into sextiles and their means $w_1, w_2, \ldots, w_6$ are computed. Then a variable
R = (w_2 - w_1)/(w_6 - w_5) is used for estimating distribution parameters, similarly as is S in the Alekseyev method.

2.1.1.3.4 Plotting of distribution functions

A plot of a fitted distribution function is the final practical objective of distribution fitting. From it, the values of annual runoff of desired probabilities of exceedance can be read and corresponding return periods computed.

Given the distribution parameters \( \bar{Q}, C_v \) and \( C_s \), a distribution function can be most easily plotted by using its so-called frequency factors \( \phi \) which are tabulated as functions of the cumulative probability \( p \) and \( C_s \).

The frequency factor is a standardized variable, \( \phi(p, C_s) = (x_p - \bar{x})/\sigma \), defining the distribution type. In other words, it is the ordinate of the distribution function of a model for the special case when the mean of the variable is zero and its standard deviation unity. From the above relation the variable with arbitrary mean and standard deviation is given by

\[ x_p = \phi(p, C_s)\sigma + \bar{x} \]  \hspace{1cm} (4)

or using normalized dimensionless variables \( k_p = x_p/\bar{x} \), by

\[ k_p = \phi(p, C_s)C_v + 1 \]  \hspace{1cm} (5)

The values of frequency factors for the gamma type with location parameter (Pearson III) are given in Table IV. Frequency factors for the power transformation of the gamma type (Kritskiy and Menkel) are given in most Russian books on hydrology (2), (10), for the log-normal type in V.T. Chow (8), for normal distribution in current statistical handbooks.

As an example, computation of two gamma distribution functions is shown in Table V for mean annual flows of Saugeen River, Walkerton, Ontario, Canada, from the 40-year period 1913-1952. Mean flow for this period was found to be \( Q = 1.059 \text{ ft}^3 \text{ s}^{-1} \), \( C_v = 0.26 \), \( C_s = 0.64 \).

When fitting the two-parameter gamma distribution (zero location parameter) we ignore the computed value of \( C_s \) and take frequency factors corresponding to \( C_s = 2C_v = 0.5 \). When fitting Pearson III distribution (three-parameter gamma) we take frequency factors corresponding to the computed value \( C_s = 0.64 \) (in both cases by interpolation from Table IV). Both theoretical distribution functions together with the empirical one are plotted in Figure 2. The three-parameter fit can be further improved on by changing the value of \( C_s \) (and thus the frequency factors) as long as the sum of squares of deviations between the empirical and theoretical curve becomes a minimum (see the preceding section).
TABLE IV

Frequency factors \( \xi(C_s, p) \) for Pearson III type distribution function

(three-parameter gamma), for \( C_v = 1.0 \)

<table>
<thead>
<tr>
<th>( C_s )</th>
<th>0.01</th>
<th>0.1</th>
<th>1.0</th>
<th>3.0</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>90</th>
<th>95</th>
<th>97</th>
<th>99.0</th>
<th>99.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>3.72</td>
<td>3.09</td>
<td>2.33</td>
<td>1.88</td>
<td>1.64</td>
<td>1.28</td>
<td>0.84</td>
<td>0.67</td>
<td>0.52</td>
<td>0.25</td>
<td>0.08</td>
<td>-0.25</td>
<td>-0.52</td>
<td>-0.68</td>
<td>-0.84</td>
<td>-1.28</td>
<td>-1.64</td>
<td>-1.88</td>
<td>-2.33</td>
<td>-3.09</td>
</tr>
<tr>
<td>0.2</td>
<td>4.16</td>
<td>3.38</td>
<td>2.48</td>
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TABLE V

Computation of a gamma distribution function (for mean annual flows of Saugeen River, Ontario, Canada) using frequency factors

(a) Two-parameter gamma distribution

\( Q = 1059 \text{ ft}^3\text{s}^{-1}, C_V = 0.26, C_s = 2C_V \pm 0.5 \)

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<th>Probability of exceedance, per cent</th>
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<td>( \phi C_V )</td>
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<td>( k_p = \phi C_V + 1 )</td>
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(b) Three-parameter (Pearson III) distribution

\( Q = 1059 \text{ ft}^3\text{s}^{-1}, C_V = 0.26, C_s = 0.64 \)

<table>
<thead>
<tr>
<th>Order of computation</th>
<th>Probability of exceedance, per cent</th>
</tr>
</thead>
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<tr>
<td>( \phi (C_s = 0.64) )</td>
<td>4.02</td>
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<tr>
<td>( \phi C_V )</td>
<td>1.047</td>
</tr>
<tr>
<td>( k_p = \phi C_V + 1 )</td>
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<tr>
<td>( Q_p = Qk_p \text{ ft}^3\text{s}^{-1} )</td>
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Figure 2 – Empirical distribution function of mean annual flows for Saugeen River, Ontario, Canada, fitted with a two-parameter and a three-parameter gamma distribution.
2.1.1.4 Time-dependent behaviour of annual runoff

To interpret runoff data correctly one has to know how annual runoff fluctuates with time. This has an obvious importance for long-term hydrological forecasting and long-range planning in water resources. The pattern of time fluctuations also has a significant bearing on the reliability of the estimated runoff parameters such as mean, \( C_v \) or \( C_s \). Finally, it profoundly affects the performance of flow regulating reservoirs and has to be taken into account in their design and operation.

There are two basic questions to be answered in this connexion: one, whether there is any trend in an annual runoff series, two, whether its fluctuations exhibit any regular pattern. Both questions are extremely difficult to answer with reasonable certainty because of the usual shortness of recorded runoff series.

If a trend is not clearly evident from the data and if there is no obvious physical reason for it occurring (for example a continuous deforestation of basin, a systematic large scale urbanization, etc.), it is advisable to regard the series as stationary even if a slight trend is suspected. In short series a seemingly apparent trend may be a result of mere coincidence or of other properties of the data. Generally a trend in the mean of annual runoff cannot be reliably detected in series shorter than 50 to 100 years, a trend in its coefficient of variation even in series longer than that.

2.1.1.4.1 Annual runoff as stationary stochastic process

If there is no trend in the sequence of annual runoff data, or if it has been removed, the series can be regarded as a stationary stochastic sequence or, which is the same, as a discrete stationary stochastic process.

The probability distribution of annual runoff, which in itself is difficult to determine as shown in section 2.1.1.3, constitutes only one characteristic of the stochastic process concerned and says nothing about its stochastic behaviour. The latter is characterized by the so-called autocorrelation function whose estimate is given by

\[
    r_k = \frac{\sum (Q_i - \bar{Q}) (Q_{i+k} - \bar{Q})}{n \sigma^2}
\]

The variable \( r_k \) is the correlation coefficient between the pairs of annual runoff values \( k \) years apart. In a finite \( n \)-term series the above summation can be carried out from \( i=1 \) to \( i=n-k \) so that with \( k \) increasing the reliability of \( r_k \) decreases.
In most annual runoff series only $r_1$, called the first serial correlation coefficient, can usually be estimated with a reasonable accuracy. This is too little to permit any inference regarding the type of the stochastic process concerned.

Similarly, as it has been with distribution models, simple models of stochastic processes are postulated for annual runoff series and fitted to actual data on the basis of $r$ whose departure from zero can be tested at various significance levels. For the 95 per cent level, the confidence limits are given by Anderson (9) as

$$
\delta_{95}(r) = \frac{-1.645 \sqrt{n-2}}{n-1}.
$$

Thus if $r_1$ and all the other $r_k$, $k = 2,3,\ldots$, are not significantly different from zero on the 95 per cent significance level, the series is considered purely random and all values of annual runoff are treated as mutually independent.

If $r_1$ is significantly different from zero, but no reliable inference can be made about $r_k$, $k = 2,3,\ldots$, the series can be fitted by a so-called first-order Markov chain whose equation is

$$
Q_i = r_1 Q_{i-1} + E_i
$$

where $E_i$ is a random element whose distribution is a function of that of $Q$. The autocorrelation function of this type of process is given as

$$
\rho_k = r_1^k.
$$

The use of higher order Markov chains or other more complex models is rarely justified for annual runoff series. Application of the theory of stochastic processes to annual runoff can be found in Kartvelishvili (10) and in Kisiel (11).

2.1.1.4.2 Effect of serial correlation on distribution parameters

Extensive analysis of time-dependent behaviour of annual runoff series was done by Yevjevich (12) who found that most of the serial correlation in annual runoff can be attributed to basin water carry-over from year to year and thus to rivers with large storage capacities in their basins. Since such basins also tend to reduce the asymmetry of annual runoff it can be expected (Klemes (3)) that large values of $r_1$ will be found together with small positive or negative values of $C_8$. Yevjevich's data seem to confirm this assumption (3).

A positive value of $r_1$, which is the most common case in annual runoff series, represents a tendency for fluctuations about the mean to perpetuate themselves so that high values tend to persist as well as do the low ones. This means
that values in the vicinity of certain $Q_i$ will not differ much from the latter, thus adding only little new information to that which the value $Q_i$ itself gives us about the variability of the series. The maximum amount of information that each value adds to that already available is in case of $r_i = 0$ when no part of information contained in a value $Q_i$ can be derived from $Q_{i+1}$ or other $Q$ since $Q_i$ is in no relation to any other member of the series.

Thus the information contained in $n$ serially correlated values is smaller than that contained in $n$ uncorrelated values. From this can be derived (see (8) section 8-III) a so-called effective length of record

$$n' = \left\{ 1 - \left[ 1 - \frac{1-r_i^2}{n(1-r_i^2)} + \frac{2r(1-r_i^n)}{n^2(1-r_i^2)} \right]^{-1} \right\}^{-1}$$

(10)

For correlated series the value $n'$ should be used instead of $n$ in calculating errors of distribution parameters.

2.1.1.5 Simulation of series of annual runoff

Because of the stochastic nature of runoff, the exact time-pattern of future fluctuations of mean annual flow cannot be predicted. The chief dilemma of water resources management consists in the fact that decisions concerning the management of an unpredictably fluctuating runoff are to lead to predictable outcomes. The only way out of this dilemma is to test a given decision on the widest spectrum of possible future runoff fluctuations thus obtaining a multitude of different outcomes from which it would be possible to assess the range in which the actual outcome is likely to be, in other words, the chance that the desired outcome will occur.

By specifying the distribution of annual runoff and the type of stochastic process describing its time behaviour, account has been taken of all the possible variants of the future runoff sequence, and it is sometimes possible to derive the above information about the outcome of a certain decision directly from the equations of the distribution and the process type. However, in many practical cases the water management decisions, policies and rules are rather complex and direct analytical solutions are not tractable. In such cases one has to synthesize a large number of possible variants of the future runoff series and try out the intended water management policy on each of them thus obtaining a number of possible outcomes from which the needed information can be extracted.

It is important, however, to be aware of the fact that by simulating synthetic series on the basis of parameters and characteristics derived from a recorded historical series of runoff, one does not create new hydrological information about that runoff but merely makes better use of the information contained in the recorded data. It follows that no simulation, no matter how sophisticated the mathematical models and computers it may employ, can increase the accuracy and
reliability of results beyond that of the original data. This has not been emphasized enough and often an impression has been created that computer simulation can solve the problem of inadequate hydrological data. The real purpose of simulation is, to a large extent, quite opposite in that it can display the inherent uncertainties of runoff records more clearly, and make it easier to derive proper consequences from their existence.

Streamflow simulation is described in detail by Hufschmidt and Fiering (13), and by Fiering (14).

2.1.1.5.1 Simulation of random sequence

Simulation of a random sequence of variates with a given probability distribution is facilitated by a relationship between any probability distribution and the uniform distribution, which can be stated as follows. If a variable $X$ has probability density $f(x)$ then the distribution of a random variable

$$ R = \int_{-\infty}^{X} f(x) \, dx $$

is uniform in the interval $(0,1)$.

Since sequences of uniformly distributed random numbers are tabulated in most statistical handbooks, can be easily generated by digital computers, or created by successively drawing cards numbered from 0 to 9, the above equation is convenient to use.

The procedure is as follows. The distribution function of annual runoff $Q$ is plotted as shown in Figure 3. A random number $R_1$ is taken, say, from a table and interpreted as a decimal fraction; for instance, a number $R = 745$ will be read as 0.745, $R = 012$ as 0.012, etc. This fraction is considered to be cumulative probability, is entered on the probability axis of the distribution function, and the value of $Q_1$ corresponding to it represents the first term of a synthetic random series (Figure 3). Then the next random number is entered on the probability axis and the next value $Q_{i+1}$ is obtained. A series of any length can be synthesized by repeating this procedure.

When using a table of random numbers, the numbers must not be taken from the table at random but along some regular pattern, say, along rows or columns.

2.1.1.5.2 Simulation of first-order Markov chain

In equation (8), the term $E$ is a random variable. Therefore, knowing the distribution function of $E$, a random sequence of $E$ can be generated in the same manner as described in the preceding paragraph. Knowing the values $E$, the corresponding values of $Q$ are obtained from equation (8).
Figure 3 - Definition sketch for simulation of a random sequence of variable \( Q \) possessing a continuous distribution (\( R_i \) is a uniformly distributed random number from interval 0-1)

Although this procedure is very simple in principle, the real problem is the determination of the distribution of \( E \). The exact shape of this distribution is known only for normally distributed \( Q \) in which case \( E \) is also normally distributed. For asymmetrically distributed \( Q \), as is the case with runoff, only an approximate
shape of the distribution of $E$ can be found. Some methods to this effect have been developed by Svanidze (15).

Since the distribution of $E$ can easily be specified for a normally distributed $Q$, the first-order Markov chain can easily be generated if the variable $Q$ can be transformed into some variable $Q'$ which is normally distributed. (Hence follows the advantage of using log-normal distribution for $Q$.) In such a case, a chain of $Q'$ is first generated and then the values of $Q'$ are converted back into $Q$. This procedure can have some effect on the parameter of $r_1$ of the synthetic series, which however, should be negligible for practical purposes.

Recently Moran (16) developed a method which can be used for simulation of first-order Markov chain if $Q$ has a gamma distribution.

2.1.2 Sub-annual fluctuations of runoff

Surface runoff inventory expressed in terms of annual runoff provides a sort of information on the abundance or lack of water on certain territory which is basic for water management at a rather high level concerned with long-term development of water resources, assessing the importance of water as a limiting factor for overall economic development of a region, or for planning water redistribution on a large scale as, for instance, is the regulation of runoff from the Great Lakes in Canada and the U.S.A., or the regulation of runoff from the Volga Basin in the U.S.S.R.

However, there are many water resources management problems for which the total runoff is of little consequence, be it small or large.

A typical case may be a low-head water power development with no or relatively small flow regulating capacity. Here the total annual electric output will depend much less on the annual volume of runoff than on its distribution throughout the year. If in a wet year most of the runoff were concentrated into several large floods of which only a fraction of the water could go through turbines, then the power output of such a year could be much lower than that of a dry year with evenly distributed runoff when most of the water could be utilized.

Another example would be water pollution in a stream with a relatively constant input of waste water where the situation would depend on the day-to-day streamflow rather than on the total annual runoff.

A similar situation could occur in connexion with an irrigation project where large annual runoff would be of little help if several decisive weeks of the vegetation season were dry.

In these and many other cases, most typically in water management under natural runoff conditions or conditions with limited possibilities of water redistribution, the main importance is the seasonal, or the day-to-day water inventory. Hydrologically, such an inventory can only be based on the knowledge of intra-year, or sub-annual, fluctuations of runoff.
Mean monthly flows, or monthly runoff volumes, serve as the main tool for describing intra-year distribution of runoff.

Given $n$ years of streamflow records, frequency distribution of runoff and its parameters can be obtained for each month in exactly the same manner as described for annual runoff.

For water resources management, as well as for an overall classification of runoff distribution in individual years, a very useful characteristic is found to be a map of intra-year frequency distribution of runoff. The latter consists of lines connecting monthly flows of equal probabilities of exceedance throughout the annual cycle. The points constituting particular lines are readily obtained from distribution functions of flows for individual months. To allow for comparisons of such maps among different rivers, it is useful to express mean monthly flows $Q_m$ in terms of the long-term mean flow $\bar{Q}$. An example of such a map is shown in Figure 4 for the Danube River in Bratislava, Czechoslovakia (17). When mean monthly flows of a particular year are mapped into such a diagram, the character of seasonal runoff fluctuations can be readily assessed. In Figure 4, mean monthly flows are shown for the year 1947 whose total annual runoff was close to average. However, it could be misleading for water management to consider that year as an average one. As seen from the map, there was very high runoff in early spring but most of the summer had extremely low flows. As a matter of fact, the year 1947 yielded catastrophically low crops in most of central Europe.

### 2.1.2.1 Simulation of monthly flow series

A series of mean monthly flows represents a much more complex stochastic process than a series of mean annual flows. The annual climatic cycle introduces a periodic component into the process which influences not only the mean flow of individual months, but all parameters of their respective distributions, the distribution types, and also the correlations between flows of successive months.

If the need for simulation arises, a reasonably representative result can be obtained by using a twelvefold linear regression model as described, for instance, by Dyck and Schramm (18) or by Fiering (14). It requires specifying the 12 distributions of monthly flows and the 12 correlation coefficients between the flows of two successive months. It also implies that conditional distributions for each month can be determined.

The procedure is as follows. The flow of the first month is chosen at random from the respective distribution (as in Figure 3). For this value the conditional distribution function of flow in the next month is found and from it the value of flow is determined again as in Figure 3 by random choice. Using this value, the conditional distribution of flow in the following month is found and the procedure is repeated.

A simpler but less theoretically justified method is a so-called method of fragments by Svanidze (15). Here first a series of mean annual flows is simulated.
Figure 4 - Map of frequencies of mean monthly flows for the Danube River in Bratislava, Czechoslovakia
in the usual manner (section 2.1.1.5) and then on each mean annual flow the intra-year fluctuations are superimposed. The models for these fluctuations, called fragments, are taken from historical flow record as normalized annual hydrographs of monthly flows. Hence, how many years of records there are, so many different flow fluctuation patterns (fragments) can be used. The fragment to be used with a given mean annual flow $Q_i$ is chosen at random (correlation between annual runoff and fragment type can also be accounted for) and its 12 dimensionless ordinates are multiplied by $Q_i$ to produce the synthetic monthly flows.

For rivers with typical two-season régime of runoff (snowmelt and glacier-fed streams) a two-season model of sub-annual fluctuations can be satisfactory for many purposes of water resources management. In the U.S.S.R., two-season models have been used for flow-regulation studies since the 1930s (19) and they have recently attracted new attention in England (20), papers 6 and 7).

2.1.2.2 Mean daily flows

Except for flood control, perhaps no water management problems require more detailed data on runoff than mean daily flows. Their direct use in water resources management is essential for such purposes as assessment of navigation season on a river, on-river water-power installations, waste water dilution, and water supply from unregulated streams. Mean daily flows represent the basic streamflow variable reported by flow measuring agencies, thus forming the basis for computation of mean monthly and annual flows.

The most important characteristic of mean daily flows, which in many cases can better serve water management purposes than the original record, is a so-called duration curve representing empirical marginal distribution function of mean daily flows.

The basic duration curve, also called absolute duration curve, is obtained by plotting in descending order mean daily flows from the entire period of record and rescaling the time base of the plot to be equal to 365.25 days. The value of flow rate corresponding to a certain number of days, $m$, on this curve then gives a mean daily flow which is exceeded, on the average, during $m$ days of a year. In other words, the absolute duration curve gives the average exceedance time of a given flow value within the annual cycle. The time scale can be replaced by probability scale so that exceedance time is then given in per cent rather than in the number of days out of 365.25.

The absolute duration curve is sometimes used for defining certain standards or normatives; thus, for instance, the flow corresponding to exceedance time 364 days, denoted as $Q_{364}$, may be regarded as the practical minimum flow, the value $Q_{355}$ as a sanitary flow or the target minimum flow, etc., as it is, for instance, in the Czechoslovak water management practice.

The main use of this curve is made, however, in assessing the average navigation period on rivers, output of hydro-power plants, efficiency of river channel improvements, water supply possibilities from unregulated streams, and sanitary conditions in streams.
The absolute duration curve has to be distinguished from the so-called average duration curve which gives an average value of mean daily flow for a specified time of exceedance. This curve is compiled by averaging flow values of given exceedances in duration curves of individual years. It is sometimes used as a hydrological characteristic of the régime of mean daily flows, but has a rather rare applicability in water management.

Absolute duration curves for specified shorter periods than one year are of great practical importance for various tasks of water management. For instance, the duration curve for the growing season as a whole or separate curves for individual months thereof have practical value for irrigation water supply, etc.

### 2.2 Extending Streamflow Records of a Single Station

In many cases the streamflow record at a particular station is short, or some data in it are missing, and the question arises as to whether the series can be lengthened or its gaps filled by making use of some available longer or complete series of streamflow or other phenomena. In principle, the answer is positive whenever a relationship exists between the two series. However, the extended series never contains the same amount of information as would be the case if all its data were genuine and the actual improvement is much smaller than the optical impression suggests.

There are basically two ways of tackling the problem depending on whether a statistical or a physical relationship between streamflow and the auxiliary phenomenon are employed. In both cases, streamflow at a station can be related either to one or to more auxiliary phenomena, for instance, to one or more streamflow records of nearby stations on the same or some other river, to one or more precipitation records, to a combination of records of streamflow, precipitation, air humidity, temperature, or even such phenomena as tree-rings, deposit layers in lakes, etc. In some cases both the statistical and deterministic approach can be used, sometimes only one of them.

#### 2.2.1 Statistical Approach

##### 2.2.1.1 Effect of Correlation on Effectiveness of Record Extension

The most common working assumption in statistical analysis of the relationship between streamflow (or runoff) and some other natural process is the linearity of this relationship. In such a case the closeness of the relationship can be measured by the coefficient of correlation, \( r \), representing the decisive parameter on which primarily depends the amount of information that can be transferred from the record of one event into that of the other one. The increase of information results in a reduction of sampling errors or parameter estimates of the extended series. Since sampling errors of all parameters are functions of the record length, the information increase of the original series can be measured by an effective increase of the latter. Langbein (21) gives the following approximate formula for calculating the so-called effective period of record,
where $N_s$ is the number of years in the short series (to be extended) and $N_{ext}$ the number of years in the extension (the difference between the long and the short series).

If $N_{eff}$ is not greater than $N_s$, the extension is ineffective; also, the shorter is $N_s$ the greater must be $r$ in order that $N_{eff} > N_s$. As an example of the extension effectiveness, if $N_s = 5$, $N_{ext} = 15$ (the long record has 20 years) and $r = 0.8$, the effective length is $N_{eff} = 7.1$ years, a gain of 2.1 years. If $r$ were 0.63, the extension would have no effect since $N_{eff} = 5$. If however, $N_s$ were equal to ten years its extension on the basis of a 20-year series would lose its effect only if $r$ dropped to 0.45.

A more precise method of treating this important aspect of statistical extension of hydrological data has been developed by Matalas and Jacobs (22) who give formulae for the error in the mean and the variance of the extended series.

2.2.1.2 Pitfalls of regression

In hydrology the least square linear regression is the most commonly used mathematical model for relating two sets of data. It consists in fitting a straight line (regression line) to the data such that the sum of squares of deviations of all empirical points from the line is minimum. There are two regression lines, each corresponding to the direction in which the deviations have been measured.

The common use of the regression model for extending streamflow record is such that the regression line of streamflow $Q$ and some variable $x$, used as extension basis, is computed from concurrent sets $Q_i, x_i, i = 1, \ldots, n$, and then unknown values $Q_j$ are read from the line for known values $x_j, j = n+1, n+2, \ldots, n+m$.

The drawback of this procedure consists in the fact that a value $Q_j$ estimated from the regression line represents the conditional average of all the possible $Q$'s for given $x_j$, while a value $Q_i$ from the original set represents one random value of all the possible $Q$'s for given $x_i$. Thus the extended series of $Q$ is not homogeneous since some of its values (the original ones) are stochastically related to $x$, while the others (those in the extension) are related to $x$ functionally through the regression equation

$$Q_j = \overline{Q} + R_{Qx}(x_j - \overline{x})$$

where $R_{Qx}$ is the regression coefficient of $Q$ on $x$. 
To make the $Q_j$'s qualitatively equivalent to the $Q_1$'s, a random component must be added so that

$$Q_j = \bar{Q} + R_{Qx}(x_j - \bar{x}) + \varepsilon_j \sigma_Q \sqrt{1-r^2} \quad (14)$$

where, in compliance with the assumptions on which the linear regression model is based, $\varepsilon_j$ is a random normal variate with zero mean and unit variance, and the term $\sigma_Q \sqrt{1-r^2}$ is the conditional standard deviation of $Q$, $r$ being the correlation coefficient between $Q$ and $x$. Such modification makes the extended series homogeneous and the parameters computed from it are unbiased, as shown by Matalas and Jacobs (22).

The random component, while improving the situation from the statistical point of view, creates some new hydrological problems.

First of all, if the purpose of the extension is to fill the gap in an existing series, we are interested in obtaining the best possible approximation of each of the missing terms separately. However, the inclusion of the random component gives us merely the best estimate of their collective mean and variance. This means, in other words, that the estimate of each particular term can be much worse than is the average given by the regression line, but that the differences, being random, will cancel out if we consider all the estimated values together. Thus if we are interested in the pattern of the series itself, the inclusion of the random component can have a distorting effect.

In some cases this dilemma can be overcome by simulation. If, for instance, the extended streamflow series is to be used for storage reservoir design, one can simulate a number of series, always retaining the original members and changing only the terms in the extension using their corresponding conditional distribution functions in the same manner as described in section 2.1.1.5.2. By this procedure one avoids the risk of relying on a single, and possibly heavily distorted, series, and at the same time preserves the stochastic relationship between the two correlated variables.

Another complicating factor which, conversely, will be best revealed when using simulation, is the assumption, built in to the least square regression model, that the conditional standard deviation of $Q$ is the same for any value of $x$. This implies the equivalence of all conditional distributions along the regression line, which is very unlikely, indeed impossible for phenomena like streamflow or precipitation. It would mean that, for example, the deviation from the regression line can be as large for the smallest values of the variables as it can be for the largest values. This could considerably distort the picture mainly in estimating small values of the variable when one could easily obtain negative values. A typical example of such a situation is shown in Figure 5 where maximum discharge is correlated with spring runoff.* Assuming the linear regression model it would be quite possible to obtain negative discharge for runoff smaller than about 30 mm or negative runoff for discharge under about 400 m$^3$ s$^{-1}$. The plot of data as well as the physics of the related phenomena suggest that their variability in small values is definitely lower than it is in high values; hence the common linear least-square regression model is inappropriate in this case.

Figure 5 - Illustration of possible distorting effects caused by the assumption of a constant conditional variance in the least-square regression model.
This difficulty can, in theory, be easily overcome by using a more sophisticated regression model: in general, by fitting a proper bivariate distribution to the two samples of concurrent events. However, this task brings with it many practical difficulties. So far, the theory is well developed only for the multivariate Gaussian distribution, but little is known about multivariate non-normal distributions. Recently Moran (16) published an approximate method for fitting a bivariate gamma distribution.

The problems described for simple regression are basically the same in multiple regression where streamflow (or some other variable) is correlated simultaneously with several other variables. There are many methods in multivariate analysis which can help in finding out the relative effectiveness of particular phenomena for use in extending hydrological records. Some of the most relevant methods like principal component analysis, factor analysis, and some other, have been described, for instance, by Stammers (23).

2.2.2 Deterministic approach

2.2.2.1 Lumped systems

A lumped-system model can, to some extent, be regarded as a deterministic counterpart to the simple regression model since it relates streamflow to only one other variable. However, while in the regression model the other variable can be any variable which parallels the time behaviour of runoff, in the deterministic model it must be a variable which is directly physically related to it being either its cause or its resultant. In other words, a deterministic model requires that there be a physically defined mechanism which transforms a variable $x$ into streamflow $Q$ or vice versa. The objective is not only to relate $x$ and $Q$ but to describe the mechanism of their relationship and to use the latter for the prediction of $Q$ from $x$.

This is a typical problem of system analysis where it is known as the "black box" problem. In system-analysis formulation, the mechanism relating $Q$ and $x$ is defined as the property of a system that transforms an input $x$ into an output $Q$, or an output $Q$ into an input $x$. Mathematically, this property is described by some system-response function.

In a lumped-system model, we are not concerned with the detailed analysis of all properties and all particular processes of the system, say of a basin that transforms precipitation $x$ into streamflow $Q$, but in the total effect, in the, so to say, macroscopic behaviour of the system.

Basically, the lumped-system can be either linear which means that the principles of proportionality and superposition hold, or non-linear where they do not. Although most hydrological systems are non-linear, they often are approximated by linear ones since non-linear systems are difficult to deal with.

A typical example of a linear lumped-system model is the unit hydrograph method where the shape of the unit hydrograph represents the system response function. Mathematically, this method can be described by the convolution integral giving runoff (as function of time $t$) as
where \( x(\tau) \) is precipitation as function of time and \( u(t-\tau) \) is the unit hydrograph.

Knowing the unit hydrograph for a given stream-gauging station, it would be theoretically possible to extend streamflow record from precipitation-excess data. However, the unit hydrograph itself changes with seasons and climatic conditions so that it can be used only for very specific conditions. Also, since it does not reflect the contribution of groundwater to streamflow, it can be effectively used only when the groundwater portion in streamflow is small, i.e. in case of floods.

Yevjevich (12) and later Frind (24) used a simplified unit hydrograph techniques to relate annual amount of runoff to that of annual total of effective precipitation.

Another typical example of the lumped-system approach is the so-called flood routing techniques which derive streamflow at one point from that at some up-stream point. In this case the system is represented by a reservoir with outlet capacity defined generally as non-linear function of storage.

2.2.2.2 Distributed systems

The larger the areal extent of the geographical unit to be modelled, the more it is likely that a lumped-system model will not yield satisfactory results. This is natural because heterogeneity of natural conditions increases with the area and, being represented by areal averages, the input and output are still more loosely related to each other. By gradually increasing the area of the analysed geographical unit one would eventually come to a size for which an input-output pair (e.g. daily precipitation and runoff totals) would appear completely unrelated.

The idea of the distributed-system model is to limit the lumping to a size for which the input-output relation is clearly evident and for which a more or less unique system response (e.g. unit hydrograph) can be specified. It then becomes possible, by combining outputs from a number of subsystems, to obtain a unique input-output relationship even for a large area for which, taken as a whole, it is obscured by the averaging process.

Obviously, the distributed system is theoretically preferable to the lumped one. There are limits, however, as to the detail to which a distributed-system model can be usefully developed. One limit arises from the fact that the input-output relationship for each element is subject to errors and, however small they may be, their combined effect can result in substantial noise if the number of elements is large. Another limit is imposed by the availability of data necessary for specifying the input-output relationships for individual elements.

The term distributed-system has sometimes been used with respect to a vertical, rather than the above-described horizontal, distribution. It means that while the whole area is considered one unit, the route from input to output is decomposed into a sequence of input-output pairs representing various stages of the system mechanism.
The Stanford watershed model* developed by Linsley and Crawford (25) is an example of such a model.

The use of sophisticated deterministic models for extension of past streamflow records is very limited since, as a rule, only inadequate and incomplete data for the concurrent periods are available.

2.3 INDIRECT DETERMINATION OF STREAMFLOW

If no streamflow records exist for a particular river site the necessity often arises to simulate the whole streamflow record. This is always a very risky undertaking since there is no basis for evaluating the representativeness of the simulated record. It is therefore strongly recommended that a stream-gauging station be established as soon as the need for streamflow data becomes known so that there be at least a short period of record on which the synthesized streamflow can be tested.

The importance of indirect determination of runoff is still increasing. One reason is the necessity for development of water resources in remote areas (where no streamflow records exist) in connexion with the pressure for economic development (mineral resources, water power). This is being felt mostly in countries like Canada or the U.S.S.R. which have vast yet undeveloped territories with no data on streamflow. A similar situation prevails in most developing countries in Africa and Asia where the development of hydrometeorological networks and the rate of acquisition of hydrological data cannot cope with the rate of economic development.

However, even in highly developed industrial countries with well established and dense hydrometeorological networks indirect determination of runoff is gaining importance. In these countries the result of the existing high degree of utilization of surface runoff distorts still more its natural régime. Under such conditions, streamflow records become still less representative, since they no longer reflect natural runoff fluctuations but merely an ever-changing man-made régime. Thus they have very limited hydrological value. It is often the case that, in spite of extensive and detailed flow records, indirectly determined streamflows must be used for water resources planning. This tendency is bound to persist so that indirect determination of runoff will remain one of the important areas of hydrology and cannot be considered as merely a necessity of a transitional period before enough records are available.

2.3.1 Hydrological analogy

Hydrological analogy employs climatic and physiographic similarities between basins to make inferences about streamflow in one basin (the design basin) from streamflow records in another one (analogue basin).

The simplest case is when the design and analogue basins significantly overlap. Such a situation frequently arises if streamflow data are needed for some river

* The Stanford watershed model has provided a basis for a company called Hydrocomp International that has been incorporated in California (591 Lytton Avenue, Palo Alto, California 94301) under the direction of Dr. Linsley and Dr. Crawford, and which specializes in deterministic simulation of streamflow and related problems.
cross-section nearby a gauging station on the same river. In such a case streamflow in the design cross-section can be approximately token as

\[ Q_d = Q_a \frac{q_d A_d}{q_a A_a} \]  

(16)

where \( q \) and \( A \) are, respectively, the specific runoff (see 2.3.2.1) and area of the analogue (a) and the design (d) basins. This technique gives reasonably good results for mean annual flows and usually also for mean monthly flows. For mean daily flows results are acceptable only if the two river cross-sections are very close to each other and there is no larger tributary between them.

The same method can be used even if the analogue and design basins do not overlap but have similar climatic, geographical, and other conditions. The area of the design basin should not differ from that of the analogue basin by more than 30 per cent to 40 per cent.

If the values of specific runoffs are not available the adjustment can be based on precipitation totals. In this instance, even if the overall similarity between the two basins (vegetal cover, altitude, physiography, etc.) is very close, the results are generally less representative.

Both lumped- and distributed-system modes derived for the analogue basin can also be used to synthesize runoff from a design basin. For lumped-system models the similarity between the two basins must be very close to yield acceptable results. Distributed models are more flexible in this respect but their applicability is usually hampered by lack of available data.

2.3.2 Regional characteristics of runoff

Rough estimates of runoff from ungauged basins can be made on the basis of regional characteristics derived from existing networks of streamflow and rainfall gauging stations and mapped in the form of isolines, expressed by empirical regional formulae, typical annual hydrographs, and other means.

2.3.2.1 Specific runoff

Specific runoff, sometimes also termed unit runoff, is defined as runoff from a unit area per unit time. For water resources inventory, the so-called average specific runoff \( q_a \) is of great importance. It is derived from the long-term mean flow by dividing it \( q_a \) by the basin area,

\[ q_a = \frac{Q}{A} \]  

(17)

and it is expressed in \( \text{l s}^{-1} \text{ km}^{-2} \), in \( \text{m}^3 \text{ s}^{-1} \text{ km}^{-2} \), or in \( \text{ft}^3 \text{ s}^{-1} \text{ mile}^{-2} \).

Values of \( q_a \) are plotted into maps and isolines are drawn from which long term average flow from ungauged basins can be derived by multiplying the interpolated \( q_a \) by the basin area.
Apart from the average specific runoff, the so-called maximum specific runoff $q_{\text{max}}$ is used in flood flow analysis. This characteristic is defined as a ratio of peak discharge of a maximum flood and basin area. Isolines of $q_{\text{max}}$ serve for estimating flood peaks.

2.3.2.2 Coefficient of runoff

Coefficient of runoff is a parameter parallel to that of specific runoff. Generally it represents runoff in per cent of precipitation over the basin. As specific runoff, it can be defined either on a long-term basis thus representing the average annual amount of runoff in per cent of the average annual precipitation total, or for maximum flows in which case it reflects runoff conditions during floods.

As a long-term characteristic it is sometimes termed effective precipitation, while the term "runoff coefficient" is reserved for the parameter derived from storm runoff and used in the so-called rational method for estimating peak flows of floods (see section 4).

In water resources inventory the former parameter is of considerable importance for estimating average annual runoff from precipitation data.

2.3.2.3 Regional pattern of runoff variability

The information about surface runoff contained in the value of mean annual flow can be substantially enhanced if the coefficient of variation of annual runoff, $C_V$, can be determined. Knowing these two parameters, and assuming some plausible type of probability distribution one has a valuable, although approximate, description of the overall conditions of surface runoff in a basin.

Coefficient of variation of annual runoff from ungauged basins is usually determined by means of regional empirical formulae derived on the basis of existing runoff records. It has been found that $C_V$ usually shows a close correlation with the basin area $A$ and its average specific runoff $q_a$. There are numerous formulae of the type $C_V = f(A,q_a)$ in Russian hydrological literature (e.g. (2)). Gladwell (26) attempted regional analysis of the mean, coefficient of variation, coefficient of asymmetry, kurtosis, and serial correlation of annual runoff for the State of Washington, U.S.A.

Intra-year runoff fluctuations can also be estimated for ungauged basins. This is usually done on the basis of typified annual hydrographs compiled for various regions from existing streamflow records. These are frequently expressed in terms of normalized monthly hydrographs. Browzin (27) and Pentland (28) carried out studies of this type for the Great Lakes basin. Pentland also produced maps of specific runoff for individual months.

Similarly, duration curves of mean daily flows can be normalized and typified for various climatic or geographical regions as documented, for instance, by Andreyanov (29) and Dub (17).
A problem often arises in water resources management to estimate probability of the combined annual or seasonal runoff of two or more rivers in a certain region, for example if a river is diverted into another river or if runoff down-stream of the confluence of two rivers is to be assessed.

The resulting distribution function is a convolution of distribution functions of the components. Because of the importance of this problem, an illustration is given below for a simple case where the distribution function of combined annual runoff of two rivers is sought. Annual runoff is denoted by \( X \) for the first river, by \( Y \) for the second, and the combined runoff (say, below the confluence of the two rivers) is denoted by \( Q \), so that \( Q = X + Y \). The respective distribution functions are denoted by \( F_X(X) \), \( F_Y(Y) \) and \( F_Q(Q) \). Let us first assume that the annual runoffs \( X \) and \( Y \) are mutually independent, i.e. that the correlation coefficient \( r_{xy} = 0 \). In such a case each value of \( X \) can be combined with each value of \( Y \), and from the general properties of runoff distributions (see section 2.1.1.3) it follows that both \( X \) and \( Y \) can take on values from 0 to \( \infty \).

Under these assumptions the convolution integral

\[
F_Q(Q) = \int_{0}^{\infty} F_X(Q-Y) \, dF_Y(Y)
\]

defines the probability of exceedance of \( Q \). The above integral can easily be evaluated either by a numerical or graphical procedure. Either procedure requires that one of the functions \( F_X \) and \( F_Y \), which are continuous, be replaced by a step function of a finite number of steps, \( m \). The corresponding discrete form of the convolution integral becomes (for \( F_Y \) replaced by a step function)

\[
F_Q(Q) = \sum_{i=1}^{m} F_X(Q = Y_i) \, \Delta F_Y(Y_i),
\]

\( 1 = 1, 2, \ldots, m \).

The graphical procedure is as follows. Functions \( F_X \) and \( F_Y \) are plotted (in linear scales for both the variable and its cumulative probability) and \( F_Y \), say, is replaced by \( m \) steps that need not necessarily be equally spaced (Figure 6 (a), 6 (b)). The other function, in the present case \( F_X \), is then superimposed on top of each of the \( m \) steps of \( F_Y \), with its probability scale multiplied by the width of the corresponding step \( \Delta F_Y \); this multiplication "compresses" the function \( F_X \) to the size of the step as shown for the \( i \)-th step in Figure 6 (c). The result of this operation (superposition of \( F_X \) with reduced probability scale on top of the \( i \)-th step \( \Delta F_Y \)) is a graphical interpretation of the mathematical statement \( F_X(Q-Y_i) \Delta F_Y(Y_i) \) and it represents the \( i \)-th partial distribution function of \( Q \).
Figure 6 - Definition sketch for graphical solution of convolution of two distribution functions
The complete function $F_Q(Q)$ is a "mixture" of all the $m$ partial functions, and is represented by the sum in equation (19a). It is obtained by adding up, for each value of $Q = Q_j$, the corresponding abscissae values of all the $m$ partial functions as indicated in Figure 6 (c). By plotting the abscissae for $n$ different values of $Q$ we in fact solve $n$ equations of the type

$$F_Q(Q_j) = \sum_{i=1}^{m} F_x(Q_j - Y_i) F_y(Y_i),$$

(19b)

$$i = 1, 2, \ldots, m$$

$$j = 1, 2, \ldots, n.$$ 

The numerical solution consists of solving these equations as a system of $n$ linear equations of $n$ unknowns. The system can be conveniently written in matrix form as

$$\begin{bmatrix} F_Q(Q_1) \\ F_Q(Q_2) \\ \vdots \\ F_Q(Q_n) \end{bmatrix} = \begin{bmatrix} F_x(Q_1-Y_1) & F_x(Q_1-Y_2) & \cdots & F_x(Q_1-Y_m) \\ F_x(Q_2-Y_1) & F_x(Q_2-Y_2) & \cdots & F_x(Q_2-Y_m) \\ \vdots & \vdots & \ddots & \vdots \\ F_x(Q_n-Y_1) & F_x(Q_n-Y_2) & \cdots & F_x(Q_n-Y_m) \end{bmatrix} \begin{bmatrix} F_y(Y_1) \\ F_y(Y_2) \\ \vdots \\ F_y(Y_m) \end{bmatrix}$$

(20)

where

$$\sum_{i=1}^{m} F_y(Y_i) = 1.$$ 

(21)

If the annual values of runoff $X$ and $Y$ are correlated the only difference in the procedure is that instead of using the original distribution function $F_x$ for each step $\Delta F_y$, now distribution functions conditional on $Y_i$ must be used.

It is also possible to calculate the mean, and the coefficients of variation and of asymmetry of $Q$ from those of $X$ and $Y$ and the correlation coefficient $r_{XY}$, and to represent $F_Q$ by a fitted distribution model. The parameters of $Q = X+Y$ are given as follows (30):
mean \[ \bar{Q} = \bar{X} + \bar{Y}, \]  
variance \[ \sigma_Q^2 = \sigma_X^2 + \sigma_Y^2 + 2r_{XY} \sigma_X \sigma_Y, \]  
coefficient of skewness \[ C_s(Q) = \frac{1}{\sigma_Q^3} \left( C_s(X) \sigma_X^3 + C_s(Y) \sigma_Y^3 + 3m_{21} + 3m_{12} \right), \]  
where \[ m_{21} = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X})^2 (Y_i - \bar{Y}), \]  
\[ m_{12} = \frac{1}{n} \sum_{i=1}^{n} (X_i - \bar{X}) (Y_i - \bar{Y})^2. \]
The notion of water deficiency is closely related to that of drought; however, there is a profound difference between them.

Drought is, in principle, a geophysical notion referring to subnormal conditions of water occurrence in some physical environment, like the atmosphere, soil or certain territory, river, etc. It does not have to classify the situation with respect to the human element, and as such does not necessarily imply a desirability of changing the situation.

Water deficiency, on the other hand, classifies the conditions of water occurrence from the point of view of human needs, thus representing a notion falling into the category of water management.

In the current usage there is a considerable overlap between the two terms and it is true that water deficiency most usually arises as a consequence of drought. However, it often happens that water deficiency in certain areas of human activities is not experienced even under conditions of quite severe droughts and, vice versa, there are frequent situations when deficiency occurs under quite normal, or above-normal, conditions of water occurrence.

3.1 DROUGHTS

3.1.1 Interpretations

The term "drought" is interpreted variously, though not conflictingly, according to the area of scientific interest, and to the degree to which this term is used as a synonym for water deficiency in various fields of human activities.

A recent (1967) WMO Report by Subrahmanyam (31) gives a concise survey of the variety of interpretations, classifications and analyses of droughts all over the world. It is obvious from this survey that drought is a relative rather than an absolute condition.

For example, precipitation drought can be defined on a meteorological basis as a period of time (from several days to several weeks) with no rain or with rain not exceeding some absolute depth (frequently 0.01 inch or 0.1 mm per day), or on a climatological basis as a period (from several weeks to the whole year) during which precipitation total is below a certain percentage (30 to 90 per cent) of the mean. Drought can also be defined as atmospheric drought related to certain conditions of air temperature, humidity, wind and precipitation, or as hydrological drought related to reduction in streamflow, sinking of lake and reservoir levels, or lowering of ground water tables.
Considering the consequences rather than the cause, one can encounter a
notion of, say, agricultural drought related to crop yield or soil moisture (wilting
point), etc. In the context of this text, definitions based on consequences for
different areas of human activities are regarded as relating to water deficiency
rather than to drought.

3.1.2 Objectives and purpose of analysis

Depending on the purpose of analysis, an appropriate definition of drought
must first be adopted and criteria selected that will best reflect the particular
aspect of subnormal water occurrence. This stage is perhaps the most important
one, and there is no general rule as to how to make the proper choice. Inevitably
the criteria used will be subjective to some extent.

Nevertheless, whatever criterion may be used, the analysis will always be
concerned with some of the following aspects of droughts (32):

(a) Duration of periods meeting the selected criterion (e.g., non-exceedance of
0.1 mm of precipitation per day);

(b) Probability of occurrence (e.g., that of drought of selected duration);

(c) Severity (e.g., total deficiency in water supply with respect to some
reference level and some duration);

(d) Time of occurrence within the annual cycle;

(e) Areal extent.

Analysis of these aspects is important in water management in a number of
respects.

Firstly, it helps to assess the general feasibility, under natural conditions,
of a given area for various socio-economic purposes, such as growing of certain plants,
recreational use of rivers, conditions of water supply for settlements, chance of
fire hazards, etc.

Secondly, it provides a basis for a quantitative assessment of risks
associated with the above-mentioned activities.

Thirdly, it provides a basis for operational water management by improving
the possibilities of drought forecasting and thus for taking appropriate counter-
measures in time.

3.1.3 Methods of analysis

Generally, methods of mathematical statistics are used for obtaining quanti-
tative characteristics of the variables concerned. These can be divided into three
groups (32):
(a) Empirical methods making inferences about properties of the variable strictly on the basis of observed data;

(b) Monte Carlo methods making use of the empirical data to derive some generalized characteristics to be used for simulation of large synthetic data samples with the intent to get more insight into the properties not adequately reflected in a short empirical sample;

(c) Analytical methods arriving at generalized characteristics of the variable by analytical means, mostly by applying the probability theory.

Of great practical interest also for forecasting purposes are relationships between droughts and other geophysical phenomena such as synoptic situations in the atmosphere and in the oceans, macroclimatic fluctuations and extraterrestrial conditions (solar activity and other), which are being investigated by both deterministic and statistical methods.

As an example of standard analysis, a recent (1970) work of E. S. Joseph (33) can be cited. It is concerned with annual hydrological droughts defined as the lowest mean discharges at a specified measuring point in a stream for 14 consecutive days during a climatic year starting 1 April. The analysis was carried out for 37 gauging stations in the Missouri river basin in the U.S.A. and its objective was to find an appropriate model for the probability distribution of the above characteristic. The author used five different distribution types (gamma, log-normal, square-root normal, and Weibull) and found that the gamma type yielded best results in the sense that it could be accepted as a good fit in the largest number of cases (35 out of 37).

The practical value of such analysis lies in the possibility of assessing the minimum fortnightly runoff for different return periods.

In (32) Yevjevich suggests an objective approach to drought analysis based on the theory of runs. A run is an uninterrupted sequence of variates that are lower or higher than some reference value. If, for instance, in a flow record a value $Q_o$ is chosen as a reference value, drought can be defined as an uninterrupted sequence of flows $Q < Q_o$. The duration of drought can be identified with the length of this sequence, $\tau$, and severity of drought $S_\tau$ can be measured by the total water deficit with respect to $Q_o$ during period $\tau$, so that $S_\tau = \int_{t_1}^{t_1+\tau} (Q_o - Q) dt$. As seen from Figure 7, for a given value of $Q_o$ both the drought durations $\tau$ and severities $S_\tau$ will vary and will thus have certain probability distributions. The latter can be found either empirically, or by Monte Carlo techniques, or analytically derived from the nature of the runoff (or other relevant) process.

3.2 WATER DEFICIENCY

Water deficiency arises if water occurrence is related to human interests or activities and if, from the point of view of these interests and activities, there is not enough water available.
Thus, for instance, the problem of water deficiency does not arise in a desert with almost no water at all, unless there is an intention to make use of it for some socio-economic purpose, as say agriculture, mining, tourism, etc.

On the other hand, severe water deficiency can arise in relatively humid regions as a result of imbalance between existing water resources and water needs of the economy.

To determine the degree of water deficiency is not the task of the hydrologist except for that part of it which arises from the hydrological properties of the measure taken to make up the deficit (evaporation and infiltration losses). Otherwise the actual extent of water deficiency must be determined by the water-management expert since, in principle, it is an economic rather than hydrological variable. Only after the degree of water deficiency has been established can hydrological methods be applied to find out how to compensate a given water deficit.

In dealing with water deficiency, several different notions can be encountered as summarized in the subsequent paragraphs.

3.2.1 Water needs

The term "water need" is usually used in a general sense to indicate the existence of situations in which water is used by man, thus making him dependent on its availability in nature.
It has no distinct quantitative connotation and is used in referring to such topics as, for instance, the future water needs of a nation in the context of its population growth, or to water needs of one type of industry as compared with other types, etc.

3.2.2 Water requirements

This term usually refers to empirically established average rates of water use for particular purposes.

In municipal water supply it is usually given in litres (gallons) per capita per day, in industrial water supply in litres (gallons) per metric ton (ton) or a cubic unit of product, in irrigation in m\(^3\) per hectare per year (or in cubic feet per second per acre during irrigation season), etc.

Detailed listing of water requirements around the world is given, for instance, in (34).

The notion of water requirement does not relate water use to cost of water although it implies certain market conditions as a result of which each particular "requirement" has evolved. It is also based on a certain type of technology and is thus by no means constant. For this reason, water requirements of the above type have a limited validity as a means for assessing future water needs and can be used only as rough guides for the not too distant future.

3.2.3 Water demands

Water demand is defined in economic terms. It can be interpreted as the amount of water that people are prepared to pay for at a particular price.

Thus, in contrast to water requirement, water demand is a much more realistic characteristic, and it is the demand that should be used in water management and planning for the assessment of actual water needs.

It is the fact that water demand responds to price charged for water which makes water deficiency, or water shortage, as distinct from drought, making it an economic rather than a hydrological entity.

3.2.4 Water use

An important aspect of water demand is the fact that the total demand in a region is not the sum of individual water demands. This is because (1) not all water uses have consumptive character and therefore (2) a large portion of water can be used simultaneously for different purposes (for instance, augmentation of low flows in a river may at the same time serve for pollution abatement, navigation, fisheries, recreation, hydro-power production along the river), and (3) a large portion of water can be used successively by different users (for instance, water used for hydro-power production in a high-head installation can serve for low-flow augmentation downstream of the dam, and at some point far downstream be finally used for irrigation).
From the hydrological point of view, and ultimately also from the point of view of water management, the most important part of water use is consumptive use which depletes the natural water potential of the basin or river.

Typical values of water consumption are as follows:

- Water power generation: 0 per cent
- Industrial use (for production): 5 per cent
- Municipal use: 5 to 10 per cent
- Irrigation: 30 to 80 per cent
- Cooling water:
  - (a) Complete recirculation (cooling towers): 100 per cent*
  - (b) No recirculation (on-stream): 3 per cent*

3.2.5 Water losses

Water losses represent a special category of water use. It is such water that results not from the final purpose of water utilization, but from the means by which water is supplied to the consumer.

Water losses can also be consumptive and non-consumptive.

(1) Consumptive losses are mainly represented by the increase in evaporation and transpiration.

A typical example is evaporation from an artificial storage reservoir created by a dam, a navigation canal, a pond. It is usually true that the evaporation from a water surface is greater than that from land, and therefore a new water surface increases consumptive use of water. However, it has been observed that in humid regions with lush vegetation evaporation from a free water surface may be smaller than that from vegetation-covered lands. Evaporation losses from a water surface represent the only component of water demand that is determined by the hydrologist. For this purpose maps of mean monthly or annual evaporation depths from water surfaces are being used that are compiled from empirical data obtained from direct measurements on existing lakes and reservoirs, sometimes supplemented by data theoretically derived by standard methods of evaporation estimates.

(2) Non-consumptive losses can increase the total water demand of one consumer but, because of their returnable character, can reduce the demand of users further downstream. A typical example is seepage through and under a dam. If the required

* Complete recirculation reduces the total requirement for cooling water to 1/10-1/20 of the quantity required for on-stream cooling. The latter is thus usually feasible only where natural water supply is abundant (on large rivers and lakes). The advantage of the non-consumptive character of on-stream cooling is often offset by its detrimental effect on water quality (e.g. thermal, radioactive pollution).
draft from a dam for power generation is D, then the reservoir has to be designed for total draft $D + \Delta D$ (where $\Delta D$ is the seepage); the discharge utilized for power generation will be $D$ but the flow down-stream of the dam, available to subsequent users, will be $D + \Delta D$, since the seepage will eventually end up back in the river.

Similar situations are encountered in irrigation where about 20 per cent of the delivered water is lost through seepage in canals and by inefficient operation (this water appears as groundwater recharge and reduces to some extent demands of consumers depending on groundwater supplies), or in navigation where water lost through lockage and leakage in upper sections reduces requirements in lower sections of the waterway.

A special type of non-consumptive loss that must be taken into account in water management is the temporary detainment of part of the water in the form of ice. This type of loss is most common in storage reservoirs where the depth of ice has to be considered as a seasonal reduction of active storage.

3.2.6 Water deficit

The difference between water available at a given place and time and the concurrent water demands (including losses) represents the actual deficit that has to be eliminated by water from:

(a) Some other geographical location;

(b) Storage accumulated during periods of water surplus.

Usually a combination of both above elements is required since it is rare that either no storage is required or that water can be stored at the same location where it will be needed later.

3.3 STREAMFLOW REGULATION BY MEANS OF STORAGE RESERVOIRS*

3.3.1 Definition and features

Streamflow regulation (streamflow control) is a purposefully performed redistribution of runoff by man-controlled storage reservoirs.

Its technical objective is to find a relationship between hydrological characteristics of (1) the natural régime of runoff, (2) the means of regulation, and (3) the resulting runoff régime.

The overall economic objective is then to select a solution that best serves certain concrete purposes. This part of the problem is of no concern to hydrology; it falls completely into the domain of water resources management and is solved by standard techniques and methods of system analysis pertaining to optimization. The role of applied hydrology is to provide data to make the optimization

* This section is based mainly on reference (35).
possible, not to come up with optimum schemes of regulation. This role is very often misunderstood and hydrological concepts of flow regulation are sometimes criticized for not offering any guidance on how to choose an optimum variant (14).

The physical precondition for streamflow regulation is the possibility to accumulate water, i.e. the availability of storage. Only through the influence of storage can a given streamflow pattern be changed into another one if there is no addition or withdrawal of water from the outside.

Natural storages of many types (lakes, swamps, glaciers, river channels, etc.) continuously influence runoff and keep changing its time pattern. However, they do not regulate it in a true sense since man has little or no control over their function.

A runoff-regulating storage reservoir must have the following features:

(a) The possibility of increasing or reducing the rate of flow to a desired level;
(b) High flexibility of operation; and
(c) Fast responsiveness to changes in operation.

3.3.2 Elements and their characteristics

The elements dealt with in streamflow control analysis are as follows:

Reservoir inflow (input), Q, represented by the natural régime of streamflow at the dam site. Mathematically it is represented by time series of flows and parameters derived from them (mean, coefficient of variation, coefficient of skewness, serial correlation coefficients, models of intra-year distribution, etc.) and generalized characteristics (type of probabilistic distribution and model of time behaviour).

Reservoir storage, S, represented by the volume of that part of reservoir storage capacity that is used for flow regulation. This part of storage is usually called the active storage to distinguish it from storage components serving other purposes (dead storage - for sediment accumulation, fire reserve and other). A typical distribution of storage in a multi-purpose reservoir is shown in Figure 8 (see also paragraph 3.3.3).

Target draft, D, is the minimum desired rate of release from the reservoir. The word desired does not refer to economical desirability and thus to an optimum rate of release. It is any minimum rate of release entered into the storage equation (see later) and its desirability refers to the fact that whatever value of draft is considered in computations, there is always some risk that it will not be possible to maintain it without interruption and the outflow will drop below the originally desired level D for some periods of time. It is often expressed as a decimal fraction of the mean inflow and called degree of regulation.
Flood limit, \( F \), is a desired upper limit of reservoir outflow which should not be exceeded as far as possible. It is a complete counterpart to reservoir draft.

Dependability of regulation, \( R \), is an index number indicating in some way the reliability that outflow will not be lower than draft or higher than the flood limit.

Release rule (operating rule) is a set of rules describing the mechanism of the reservoir regulating function. In general, it sets certain limits on release (output), mainly with regard to the amount of water in a reservoir at a given instant and to the time of the year. A typical release rule is shown in Figure 9 (39).

Reservoir outflow (output), \( Q' \), is the actual rate of reservoir release as a function of time. If seepage through and under the dam is regarded as part of reservoir release, then streamflow immediately downstream of the dam is identical with reservoir outflow and is described in a similar manner as reservoir inflow.

### 3.3.3 Types of streamflow regulation

There are two basic types of regulation depending on whether the purpose of the reservoir is to stabilize some lower limit of outflow (maintain certain draft), or some upper limit (maintain certain degree of flood protection). In the first case we are concerned with low-flow control, in the second — to be discussed in section 4 — with flood control.

These two types of flow regulation have to be treated separately, although being physically inseparable (to be able to increase flow during certain periods, one has to accumulate water and thus reduce flow in other periods, and vice versa). The reason for this is that each of these two functions of a reservoir is performed more or less independently of the other and different parts of the active storage are assigned to them (Figure 8). Such an arrangement is necessary since the basic tendency in low-flow regulation is to have the reservoir full (to have enough water for periods of shortage), while in flood control the tendency is to have the reservoir empty (to have enough space for flood detention).

It should be pointed out that a combined operation for low-flow augmentation and flood reduction is possible under certain circumstances. To discuss this topic is, however, beyond the scope of this report.

### 3.3.4 Low-flow regulation

#### 3.3.4.1 The storage equation

The basic hydrological problem of low-flow control is to find a mathematical relationship between reservoir storage \( S \), draft \( D \), and dependability \( R \). This relationship is called the storage equation and its solution may be required in any of the three possible forms

\[
S = f_S(D, R), \quad D = f_D(R, S), \quad R = f_R(D, S).
\]  

(26)
The solution of this equation is dependent on the properties of reservoir inflow $Q$ and on the release rule adopted. These two elements enter the storage equation in such a way that the continuity equation in the form

$$ (Q_i - Q'_i) \Delta t = \Delta S_i $$

is solved sequentially for a given streamflow series $Q_1, Q_2, ..., Q_m$ based on some interval $\Delta t$ (day, month, year) and subject to constraints represented by draft $D$ and the release rule.
Figure 9 - Typical release rule for a multi-purpose storage reservoir. Release from reservoir is regulated according to the zone (circled numbers) in which the actual amount of water in reservoir is found at particular times of year. (D - draft, \( Q_T \) - turbine capacity, \( F \) - flood limit, \( Q_{B+T+S} \) - combined capacity of bottom outlet, turbines, and spillway)
The approach to the solution can be either deterministic using the historical streamflow record as reservoir input, or stochastic based on a mathematical model of the input process.

3.3.4.2 Characteristics of dependability

Dependability \( R \) can be characterized in three different ways following from three different aspects of a failure in maintaining the desired draft.

Firstly, one may be interested in the frequency of occurrence of such failures, or what has become more frequent in flow regulation, in frequency of occurrence of years in which at least one failure has occurred (failure years). The relevant parameter is then called occurrence-based dependability (certainty, guarantee, reliability) and is given as

\[
R_o = \frac{n-m}{n} \times 100 \text{ per cent} \tag{28}
\]

where \( m \) is the number of failure years and \( n \) is the total number of years considered.

Often a complementary characteristic is used called risk of failure and defined as \( R' = 100 - R_o = (m/n) \times 100 \text{ per cent} \).

Secondly, one may be interested in the total duration of failures within the whole period of reservoir operation. The relevant parameter is the so-called time-based dependability defined as

\[
R_t = (1 - \frac{1}{T} \sum \Delta T) \times 100 \text{ per cent} \tag{29}
\]

where \( T \) is the length of the whole period of reservoir operation, and \( \Delta T \) is the duration of a single failure period.

Thirdly, the characteristic most relevant to water management and economic analysis of a given scheme of low-flow regulation is the quantity-based dependability which is the ratio of the actual amount of water delivered to the amount that would be delivered if no failures occurred. It is given as

\[
R_q = \left(1 - \frac{1}{T_D} \sum \Delta W\right) \times 100 \text{ per cent} \tag{30}
\]

where \( \Delta W \) is the quantity of water not delivered during a single failure period.

Generally, it holds that \( R_o < R_t < R_q \); only if draft exhibits large seasonal fluctuations may it happen that \( R_t > R_q \). If the value of one of the parameters is 100 per cent, all three are equal to 100 per cent.
It has been observed that dependabilities associated with economically optimal solutions of water supply tend to concentrate within rather narrow limits varying according to the type of water use. Typical optimum values of occurrence-based dependability, called dependability standards, are as follows:

<table>
<thead>
<tr>
<th>Type of Water Supply</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Municipal water supply</td>
<td>99%</td>
</tr>
<tr>
<td>Industrial water supply</td>
<td>95-98%</td>
</tr>
<tr>
<td>Irrigation water supply</td>
<td></td>
</tr>
<tr>
<td>(sub-humid climate)</td>
<td>70-85%</td>
</tr>
<tr>
<td>(arid climate)</td>
<td>80-95%</td>
</tr>
</tbody>
</table>

These parameters have great practical importance in water resources management and planning for preliminary estimates of storage capacities, as well as in cases when economic data for detailed optimization analysis of reservoirs are not available.

### 3.3.4.3 Target draft

As the desired rate of outflow, target draft is a discharge necessary to meet all water demands to be covered by a given storage reservoir. This means that it is the sum of direct withdrawals from the reservoir (by users along the reservoir shore), losses due to evaporation and seepage, and the minimum desired reservoir release.

It is important for the solution of the storage equation to know how target draft varies with time, since this variation influences the complexity of solution.

Uniform or constant draft represents the simplest case. Strictly speaking, it occurs rarely in practice but there are many cases where draft fluctuations are relatively small and can be neglected for the purpose of the solution of the storage equation. Typical examples are municipal water supply, supply for industry with constant rate of production, maintenance of minimum sanitary flow in a river.

Periodic draft varies with yearly seasons but its annual average remains constant. Because it is uniform in the time scale of years it is sometimes referred to as semi-uniform; typical examples: irrigation water supply in an arid climate where practically all water needed must be supplied from an artificial source, power generation for municipalities (increased load during the winter).

Stochastic draft fluctuates irregularly about a constant long-term mean; typical examples: supplementary irrigation in sub-humid or humid climate (water demand varies according to the amount of rain), compensatory flow regulation in general (reservoir release is equal to instant deficit by which some natural water supply is lower than demand), flow regulation for constant electric output of hydro-power plant with variable head on turbines.

### 3.3.4.4 Deterministic methods of solving the storage equation

In these methods the process of reservoir input is represented by a streamflow record from some past period of time. The solution of the storage equation
obtained on this basis is thus valid for this past period and any extrapolation in
time is rather hazardous. However, it has been standard practice in water resources
management to design storage reservoirs on this basis, in particular to calculate
storage capacity for a given draft so that the dependability be 100 per cent within
the underlying historical period $T$, called the "design period". Permanent value
of deterministic methods lies in the fact that they offer a good insight into the
problem and help to understand the physical mechanism of streamflow control employed
in all other methods.

The solution can be found either numerically or by means of a graphical

3.3.4.4.1 Numerical technique usually employs equation (27) in a simplified form,

$$Q - Q' = \Delta Q,$$

where $\Delta Q = \Delta S / \Delta t$, which has the advantage of having all the variables in the
same units ($\Delta Q$ is the discharge that would fill storage $\Delta S$ in time $\Delta t$).

The numerical technique offers straightforward solutions of the following
situations:

(a) Determination of storage for given draft and 100 per cent dependability,
i.e. solution of equation $S = f_S(D, R = 100 \text{ per cent})$. The solution
exists if the draft is equal to or less than mean inflow $\bar{Q}$ and it is
obtained as the maximum depletion in an infinitely deep reservoir which
is full at the beginning of design period and operates for two consecutive
design periods.

The procedure is indicated in Table VI where draft is taken as a
periodic function of time. The time unit used is one "average" month
equivalent to $\Delta t = 2.63 \times 10^6$ s. The required storage capacity is equal
to the maximum absolute value of the sum $\Sigma \Delta Q$ multiplied by $\Delta t$.

(b) Determination of dependability for given values of storage and draft,
i.e. solution of equation $R = f_R(S, D)$ for any one of the parameters
$R_o$, $R_t$, $R_q$.

The computation scheme is similar to that in Table VI with the only
difference that now the absolute value of $\Sigma \Delta Q$ cannot exceed the
value $S / \Delta t$. This limit causes drop of outflow below the value of
draft after depletion $S / \Delta t$ has been reached, thus giving rise to
failure periods. Their length, corresponding quantity of non-delivered
water, and number of failure years are used for finding relevant
dependability characteristics.

The procedure is indicated in Table VII.
TABLE VI

Example of computation of storage $S$ given draft $D$ (column 4) and dependability $R = 100$ per cent.

<table>
<thead>
<tr>
<th>Year</th>
<th>Month</th>
<th>Inflow $Q_l$ m$^3$s$^{-1}$</th>
<th>Draft $D$ m$^3$h$^{-1}$</th>
<th>Outflow $Q'_l$ m$^3$s$^{-1}$</th>
<th>Instant change in storage $\Delta Q = Q - Q'_l$ m$^3$s$^{-1}$</th>
<th>Instant storage depletion $\Sigma \Delta Q_l$ m$^3$s$^{-1}$</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>1931 I</td>
<td>12.2</td>
<td>12.0</td>
<td>12.2</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>Full reservoir</td>
</tr>
<tr>
<td>II</td>
<td>7.9</td>
<td>12.0</td>
<td>12.0</td>
<td>-4.1</td>
<td>-4.1</td>
<td>-4.1</td>
<td>First cycle</td>
</tr>
<tr>
<td>III</td>
<td>41.8</td>
<td>12.0</td>
<td>37.7</td>
<td>+4.1</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>63.5</td>
<td>17.0</td>
<td>63.5</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>35.1</td>
<td>23.0</td>
<td>35.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>VI</td>
<td>20.3</td>
<td>28.0</td>
<td>28.0</td>
<td>-7.7</td>
<td>-7.7</td>
<td>-7.7</td>
<td></td>
</tr>
<tr>
<td>VII</td>
<td>15.0</td>
<td>30.0</td>
<td>30.0</td>
<td>-15.0</td>
<td>-22.7</td>
<td>-22.7</td>
<td></td>
</tr>
<tr>
<td>VIII</td>
<td>16.7</td>
<td>25.0</td>
<td>25.0</td>
<td>-8.3</td>
<td>-31.0</td>
<td>-31.0</td>
<td></td>
</tr>
<tr>
<td>IX</td>
<td>8.4</td>
<td>15.0</td>
<td>15.0</td>
<td>-6.6</td>
<td>-37.6</td>
<td>-37.6</td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>10.4</td>
<td>13.0</td>
<td>13.0</td>
<td>-2.6</td>
<td>-40.2</td>
<td>-40.2</td>
<td></td>
</tr>
<tr>
<td>XI</td>
<td>28.6</td>
<td>12.0</td>
<td>12.0</td>
<td>+16.6</td>
<td>-23.6</td>
<td>-23.6</td>
<td></td>
</tr>
<tr>
<td>XII</td>
<td>20.5</td>
<td>12.0</td>
<td>12.0</td>
<td>+8.5</td>
<td>-15.1</td>
<td>-15.1</td>
<td></td>
</tr>
<tr>
<td>1932 I</td>
<td>13.7</td>
<td>12.0</td>
<td>12.0</td>
<td>+1.7</td>
<td>-13.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
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<td>...</td>
<td></td>
</tr>
<tr>
<td>1970 XII</td>
<td>12.8</td>
<td>12.0</td>
<td>12.0</td>
<td>+0.8</td>
<td>-37.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1931 I</td>
<td>12.2</td>
<td>12.0</td>
<td>12.0</td>
<td>+0.2</td>
<td>-37.6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>II</td>
<td>7.9</td>
<td>12.0</td>
<td>12.0</td>
<td>-4.1</td>
<td>-41.7</td>
<td>-41.7</td>
<td></td>
</tr>
<tr>
<td>III</td>
<td>41.8</td>
<td>12.0</td>
<td>12.0</td>
<td>+29.8</td>
<td>-12.7</td>
<td>-12.7</td>
<td></td>
</tr>
<tr>
<td>IV</td>
<td>63.5</td>
<td>17.0</td>
<td>50.8</td>
<td>+12.7</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>

Provided there is no number in column 7 with absolute value greater than 41.7 then $S = 41.7$ m$^3$s$^{-1} \times 2.63 \times 10^6$ s

(average month) = $109.7 \times 10^6$ m$^3$.

* All further computations are the same as in the first cycle.
Example of computation of dependability for draft D (column 4) and storage $S = 100.0 \times 10^6 \text{m}^3$. (If the "average month" is employed, then $\max |\Sigma \Delta Q| = 100/2.63 = 38.0 \text{ m}^3/\text{month}$)

| Year | Month | Inflow $Q$ | Draft $D$ | Cutout $Q'$ | Instant change in storage $\Delta Q = Q - Q'$ | Instant storage $\Sigma Q$ | Failures in water supply | Length of failure periods | Deficit $\Delta W = D - Q'$ | Failure detected by $F$
<table>
<thead>
<tr>
<th></th>
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$\Sigma \Delta T, \Sigma \Delta W, \Sigma m = \Sigma F$
The remaining two problems, \( S = f_S(D,R \neq 100 \text{ per cent}) \) and 
\( D = f_D(S,R) \) can be solved by successive approximation using 
the equation \( R = f_R(S,D) \) where \( S \) or \( D \) are kept changing 
until the prescribed value of \( R \) is reached.

3.3.4.4.2 Graphical technique employs the so-called mass curve of flow which is 
a plot of the function

\[
X(t) = \sum_0^t Q \Delta t.
\]

The reason for employing such a curve is apparent if one realizes that the sequence 
of numbers in column 7 of Table VI, from which the storage is found, would in fact 
be a mass curve of values \( (Q - Q') \) if plotted against time. In other words, the 
reason for using the mass curve is the fact that it represents an integral of the 
continuity equation (27) on which the solution of the storage equation is based.

It is technically more advantageous to use the mass curve of the 
deviation of flows from the mean,

\[
Y(t) = \sum_0^t (Q - \bar{Q}) \Delta t, \tag{33}
\]

known as the "residual" mass curve, rather than equation (32). The residual mass 
curve in fact represents storage fluctuations in an infinite reservoir if output 
equal to the draft in this case) is equal to mean flow \( \bar{Q} \).

The mass difference between two points that are \( \Delta t \) apart is given as

\[
Y(t+\Delta t) - Y(t) = (Q - \bar{Q}) \Delta t,
\]

\( Q \) being the discharge between times \( t \) and \( t+\Delta t \). It follows that the slope of 
the line connecting the two points, given by

\[
\tan \alpha = \frac{Y(t+\Delta t) - Y(t)}{\Delta t} = Q - \bar{Q}, \tag{34}
\]

has the dimension of discharge. For \( Q = \bar{Q} \) we have \( \tan \alpha = 0 \), for \( Q < \bar{Q} \) it 
is negative.

This means that, on the residual mass curve, the horizontal direction 
corresponds to average discharge, an ascending slope to above-average flow and 
a descending slope to below-average flow.

From equation (34) a slope (radial) scale for discharge can be found which 
is indispensable for graphical operations on the mass curve. The scale is obtained 
in such a manner that for a given set of discharges \( Q_i \) the values \( \Delta Y_i = 
(Q_i - \bar{Q})x k \Delta t \) are plotted as ordinates corresponding to abscissa \( k \Delta t \) (k is an 
additional constant used to magnify the scale for easy practical use). The line 
connecting the origin with point \( \Delta Y_i \) represents then the slope defining 
discharge \( Q_i \) (see Figure 10).
Figure 10 - Illustration of the use of residual mass curve for finding storage capacity for a given draft and 100 per cent dependability

The following problems have direct solutions on the residual mass curve:

(a) $S = f_S(D, R = 100 \text{ per cent})$. Assuming, for the sake of simplicity, a constant draft $D$, tangents parallel to $D$ are drawn from above to residual mass curve as indicated in Figure 10. If the last tangent intersects the vertical axis at the end of the design period in point $A$, the ordinate of the latter is transferred to the origin and the tangent drawn from the point $B$ so obtained overrules any other tangent below it (this takes care of taking into account the "second cycle" discussed in paragraph 3.3.4.4). The largest vertical distance between the mass curve (describing inflow $Q$) and the tangent (draft $D$) is the storage capacity $S$ required for 100 per cent dependability within the design period.
Figure 11 - Illustration of the use of residual mass curve for finding dependability (failure periods and resulting water deficits) corresponding to given values of storage and draft, and for finding draft for given storage and a 100 per cent dependability

(b) $R = f_R(S,D)$. An equidistant $Q^*$ is drawn to the mass curve $Q$ at a distance specifying the given storage capacity $S$. Tangents parallel to draft $D$ are drawn sequentially to both mass curves from within the strip enclosed by them, as shown by heavy lines in Figure 11 (correction with respect to the second cycle is obtained analogically as under (a)). The horizontal distances between the termination point of one and starting point of the following tangent to the upper mass curve $Q^*$ represent failure periods, $\Delta T$, while the vertical distances show the corresponding water deficits $\Delta W$. From these the required dependability parameters can be found.

(c) $D = f_D(S,R = 100 \text{ per cent})$. Path of the shortest length is drawn between the two ends of the strip (dashed line in Figure 11) and the lowest discharge along this path represents the required draft (sometimes called method of "pulled string" because of the physical resemblance of the path to a string pulled through the strip.

All other problems must be solved by successive approximation.
3.3.4.5 Stochastic methods of solving the storage equation

3.3.4.5.1 Simulation techniques

A long (at least 500 to 1000 years) synthetic streamflow series is simulated by means of methods described in section 2.1.2.1.1 and the storage equation is then solved, using this series, in exactly the same manner as if the series represented a historical flow record. Thus the actual computational techniques are those of deterministic methods, the stochastic element being represented by the nature of the underlying streamflow series.

The advantage of simulation techniques is their flexibility in allowing for taking into account any complexities of reservoir operation, including systems of several reservoirs. The problem here is not in solving the storage equation but in setting up an adequate model of the inflow process.

3.3.4.5.2 Analytical methods do not require a concrete realization of the inflow process. They proceed to the solution of the storage equation directly from the distribution and the model of time-dependent behaviour of the process of input.

At the present time direct analytical solutions exist for only a rather limited range of situations involving some special types of inflow distributions, random or first-order Markovian behaviour of inflow, and draft either constant or linearly dependent on storage. More than for solving practical problems of storage reservoir design, these methods are of theoretical importance. Their advances were first summarized by Moran in 1959 (36), the more recent advances by Prabhu in 1964 (37) and Karlvelishvili in 1967 (10).

3.3.4.5.3 Numerical techniques are based on numerical representation of the distribution of inflow and have been developed for random and first-order Markovian input (36, 38, 39). Their principle will be explained using the solution by Moran of the storage equation in the form \( R = f(D, S) \), if the inflow process is random and draft constant. In this form it can be utilized for finding the long-term storage \( S_L \) (see paragraph 3.3.4.6) on the basis of the distribution of mean annual flows.

Let the distribution function of input be represented by that of annual runoff as shown in Figure 12(a) (its analytical formulation does not have to be known) and let \( D \) represent the annual amount of water demand (rate of draft times the number of seconds per year).

It can be assumed that if \( Q > D \), the quantity \( D \) goes directly to the consumer, by-passing the reservoir where only the surplus \( Q - D \) is stored. Similarly, if \( Q < D \), the whole inflow by-passes the reservoir from which only the...
difference \( D-Q \) is added to supplement it. Thus only a quantity \( X = Q-D \), ("net inflow", can be positive or negative) causes the fluctuations in storage; its distribution function is shown in Figure 12(b).

A failure (in this case identical with a "failure year") occurs whenever there is less water in storage than is the deficit \(-X\) to be supplied from the reservoir. Given the distribution function \( F_x(X) \) and the amount of stored water at the beginning of the year, \( V_j \), the distribution of water amounts in the reservoir at the end of the year can be obtained for any \( 0 \leq V_j \leq S \) as \( F_x(X+V_j) \) truncated at zero and \( S \).

By representing storage capacity \( S \) by \( n \) discrete states \( V_1, V_2, \ldots, V_n \), and associating a layer of storage \( \Delta V \) with each of them (Figure 12(c)*), probabilities \( P_{ij} \), \( i = 1, 2, \ldots, n \), can be found from \( F_x(X+V_j) \) that final water contents of reservoir will be within the layer \( \Delta V_i \) if the initial contents were \( V_j \) (Figure 12(d)).

The complete set of conditional probabilities \( P_{ij} \) is called the matrix of transition probabilities (each term represents the probability of storage transition from state \( V_j \) to \( V_i \)) and can be found for any theoretical or empirical distribution function.

Using this matrix, steady state probabilities \( P_i \) of all reservoir states \( V_i \) can be found from the following set of equations:

\[
\begin{align*}
P_1 &= P_1 P_{11} + P_2 P_{12} + \cdots + P_n P_{1n}, \\
P_2 &= P_1 P_{21} + P_2 P_{22} + \cdots + P_n P_{2n}, \\
& \vdots \\
P_n &= P_1 P_{n1} + P_2 P_{n2} + \cdots + P_n P_{nn},
\end{align*}
\]

\( (35) \)

* In Figure 12(c) \( \Delta V_1 = \Delta V_n = 0 \); this is an arbitrary choice made for computational convenience.
Figure 12 - Definition sketch for Moran's method of numerical solution of storage equation
Figure 13 - Definition sketch showing long-term (S) and seasonal (S*) storage (X, Y - years with above- and below-average mean flow, respectively; Q - residual mass curve for mean monthly flows).
(c) Storage equation

Dependability \( R_O \) \( \Rightarrow \) from 70 per cent to 99 per cent,
Degree of regulation \( D \) \( \Rightarrow \) from 0.1 to 1.0,
Storage coefficient \( \beta \) \( \Rightarrow \) from 0 to 2.0-3.0.

An example of Svanidze's charts is shown in Figure 14.

Figure 14 - Example of a chart with pre-computed solutions of storage equation for gamma distributed reservoir input and constant draft. (D - degree of regulation; \( \beta \) - coefficient of storage, \( R_O \) - occurrence-based dependability, \( C_v \) - coefficient of variation of input, \( C_s \) - coefficient of asymmetry of input, \( r_1 \) - first serial correlation coefficient of input series). Reproduced from Svanidze (15)
\[ S = S_L + S_S \]
\[ S_S = \Delta S_S^{(1)} + \Delta S_S^{(2)} \]

Figure 13 - Definition sketch showing long-term (S_L) and seasonal (S_S) storage. (X, Y - years with above- and below-average mean flow, respectively; Q_r - residual mass curve for mean annual flows; Q - residual mass curve for mean monthly flows)
(c) Storage equation

Dependability \( R_0 \rightarrow \) from 70 per cent to 99 per cent,

Degree of regulation \( D \rightarrow \) from 0.1 to 1.0,

Storage coefficient \( \beta \rightarrow \) from 0 to 2.0-3.0.

An example of Svanidze's charts is shown in Figure 14.

Figure 14 - Example of a chart with pre-computed solutions of storage equation for gamma distributed reservoir input and constant draft. (D - degree of regulation, \( \beta \) - coefficient of storage, \( R_0 \) - occurrence-based dependability, \( C_v \) - coefficient of variation of input, \( C_s \) - coefficient of asymmetry of input, \( r_1 \) - first serial correlation coefficient of input series). Reproduced from Svanidze (15)
Similar charts were compiled for the ordinary gamma distribution by Pleshkov (see reference (19)), and for log-normal, normal and Weibull distributions by Hardison (42).

The use of such nomographs is, however, limited to the determination of long-term storage $S_L$ and the advantage of using them is to some extent offset by the necessity of finding the component $S_S$ and combining the two into the required value of $S$.

By and large, for practical use in computerized reservoir design, simulation techniques or stochastic numerical methods are preferable.

However, one important aspect of pre-computed solutions is the possibility of assessing the relative influence of individual input parameters on the solution of the storage equation.

The charts also facilitate an easy assessment of the influence of a change or error in one parameter of the storage equation on the remaining two.

This has considerable practical importance, for it allows the consequences in engineering design of errors and uncertainties in hydrological data to be seen (see section 2.1.1).

3.3.4.8 Effect of time horizon on low-flow regulation

The unique relationship among the three variables of the storage equation is valid for steady-state conditions of reservoir operation over a very long period of time (in theory, the period should be of an infinite length).

Recently, attention has been drawn to the fact that in practice the periods of time during which a dam operates under more or less stable conditions are relatively short, usually not more than about 20 to 40 years (14), (43). The question then arises as to what extent this may affect the solution of the storage equation.

Given any two of the three variables involved $(R,S,D)$, the third one will no longer be represented by a single value but will possess a distribution. For example, the dependability of regulation by a reservoir with a given storage capacity and draft will not be the same within every, say, 20-year period, and the shorter the period the greater the differences that can be expected. Only as the period length approaches infinity will the dependability converge to a firm value.

This makes a great difference in water management practice. For instance, take a case with $R_o = 90$ per cent for given draft and storage, which means that ten failure years can be expected during the next century. From the theoretical point
of view it is irrelevant when the failure years will occur; as long as there will be
ten of them within a 100-year period the 90 per cent dependability will be met.

From the practical water management viewpoint, however, it matters a great
deal whether the ten failure years will occur during, say, the first 20 years of
operation or during the last 20 years. The first case may mean an economic disaster
while the economic consequences of the second case are so remote that their present
importance is effectively zero. It is almost sure that after 80 years the dam will
not operate under the same conditions any more, may serve for an entirely different
purpose, or may even no longer be in existence.

The question of practical importance then is one of the chance that the
design value of dependability (which is a result of some economic optimization of
the whole water supply scheme) will materialize during the early years of operation;
because only then is it important for the achievement of the desired economic goal.
Questions of this sort are dealt with, for instance, in (44).
HYDROLOGICAL PROBLEMS ARISING FROM WATER EXCESS

Water excess is a water-management notion referring to the presence of more water than is desirable at a certain location, from the point of view of human needs. It is the counterpart to the term water deficiency and, as well as the latter, it can refer to situations that are hydrologically normal and do not involve any extreme water input. For instance, a swamp may exhibit water excess even under conditions of a hydrological drought if the depth of groundwater table is judged from the point of view of urbanization of the area. This type of water excess is usually a permanent one and its control does not raise any special hydrological problems. The hydrological information needed in connexion with such conditions falls into the category discussed in section 2 of this report.

The type of water excess to be dealt with in this section is one which is temporary and is a result of an extreme hydrological situation known as flood.

This type of water excess does not differ from the former one except that it is temporary. Its other specific features are its limited predictability in time, sudden occurrence, and its often catastrophic consequences.

4.1 FLOODS

4.1.1 Terminology

A flood is basically defined as a flow that overtops the natural or artificial banks of a stream. As such, a "flood" is not a purely hydrological notion but rather a geomorphological and water-management one since it involves the parameter of bank capacity governed by a complex interaction of many elements of which hydrology is only one, the others (often more important) being geology, topography, geophysical forces, and man-made changes of the environment.

In hydrological analysis the term flood had been generally used to denote a discharge wave characterized by a certain peak discharge and, eventually, this peak discharge itself. Whether a wave (or a peak discharge) will or will not cause a real flood depends on whether or not it will overtop the banks of a stream.

In this section the term flood will be used in the latter sense, i.e. to denote a discharge wave irrespective of its flooding effects.

4.1.2 Features of flood

From the water-management point of view, the main feature of a flood is the degree of its interference with man's activities, measured by the extent of actual or potential economic loss and/or by danger to human lives.
For a given river reach with a given type of land use along the river, the harmfulness of a flood generally depends on its "magnitude". Although seemingly intuitively clear, the term flood magnitude has in fact a very vague meaning since the magnitude can relate to several physical features of the flood wave. They are as follows.

**Flood elevation** is the maximum elevation of water level encountered during the flood. Theoretically, it is the most important characteristic since it decides whether the banks will be overtopped. Hydrologically, this characteristic is not the most suitable one for flood evaluation since it varies from point to point and is a characteristic of a given river cross-section rather than one of the flood wave moving along the river. Also, flood elevation is not always the best indicator of flood harmfulness. For example, a flood caused by a summer storm may be very high but often is short in duration and its volume is relatively small. As a result the overtopping of the banks need not cause much damage because of the small quantity of water that caused the flooding and because of the shortness of time during which the flooded area was under water.

**Flood discharge** is the maximum rate of flow during the flood. It is a very convenient hydrological characteristic for it relates to the flood wave itself, not to a particular river cross-section. Although it does not remain constant as the flood moves along the river, its changes are relatively small within river reaches with no large tributaries, and are relatively smooth. Nor is flood discharge affected by local changes in the river bed (erosion, sedimentation) as is flood elevation, thus being a more representative characteristic not only for the flood itself but also for any particular river cross-section. Another advantage is that the peak discharge of a flood closely coincides with the occurrence of maximum water level in the river thus being a good indication of flood elevation. In reality the peak discharge occurs before the maximum water level is reached but in most instances the difference between the maximum water level and that corresponding to the peak discharge is not significant.

The usefulness of flood discharge as a parameter measuring the harmfulness of the flood is the same as that of flood elevation. In fact, it can be used for this purpose only through its unique relationship in a given cross-section to flood elevation. In general, it is a good indication in cases where most of the flood damage is caused by the bank overtopping itself, as is the case, for instance, with earth-filled levees, road embankments, etc.

**Flood volume** is the volume of water in a flood wave above a given discharge limit $Q_i$. It is characterized by the so-called "volume curve" of flood given as

$$V_i = \int_{Q_{\text{max}}}^{Q_i} t \, dQ$$

(38)

It can easily be obtained from a flood hydrograph (see Figure 15) and represents an indispensable characteristic wherever flood damage depends on the quantity of flooding water or in cases of flood control aimed at a reduction of flood peak.
Flood Hydrograph

\[ Q (m^3/sec) \]

\[ Q_{\text{max}} \]

\[ Q_i \]

[\( dQ \) shaded area]

[t time]

Volume Curve

\[ Q (m^3/sec) \]

\[ Q_{\text{max}} \]

\[ Q_i \]

[\( V_i \) time]

\[ V (m^3) \]

Figure 15 - Definition sketch for derivation of the volume curve of flood

From the point of view of flood protection the flood volume is by far the most important flood characteristic. In spite of its importance it has not yet attracted adequate interest and the bulk of the present analyses of floods is concerned with flood discharge rather than flood volume.

Flood duration is the period of time during which discharge does not drop below a certain limit. In many cases this characteristic is closely related to the danger that the flood represents. Long duration of relatively high water levels can have a detrimental effect on stability of levees (this was, for instance, the major cause of the catastrophic floods on the River Danube in 1965) and can cause a significant rise of groundwater table over large areas with harmful consequences for agriculture, forestry and residential areas in the affected region.

Seasonal occurrence of floods has a main bearing on the land use of flood plains. A flood during the spring or early summer may be beneficial in some cases because of the fertilizing effect of suspended sediments and minerals it carries, but can result in an economic disaster if it occurs before or during the harvest. Similarly, floods on forest-covered flood plains can have catastrophic effects on wildlife if they occur early in the winter so that the water freezes and the area remains ice-covered for a prolonged period thus cutting off animals from food supply and shelter on the ground.
Flow velocity during flood may be the cause of extensive damage to structures like bridges, culverts, river training works, coffer dams, etc. High flow velocity during floods also adversely affects river navigation and dredging, and by increasing the rate of sediment transport it causes undesirable changes in river channel morphology.

4.1.3 Objectives of analysis

The ultimate objective of flood analysis is to assess the degree and frequency of interference of floods with normal life and activities of a community in order that adequate preventive measures can be taken.

Since adverse effects of a flood can be of various types and each of them may be associated with different features of a flood, the entity subject to analysis is usually not the flood as a whole but particular aspects of it, namely those having the closest relation to the particular type of danger.

To simplify the analysis it is usually assumed that most adverse effects of floods can be reasonably well related to their peak discharges which thus have become the most frequent subject of analysis.

The main objective of flood analysis is then reduced to finding the frequency of occurrence of a flood with a given peak discharge.

Similarly, if a peak discharge does not characterize the flood adequately for a given purpose, frequencies of flood volume above fixed discharge levels or of flood duration at various discharge levels are sought.

Given the usual shortness of flow records, extrapolation of frequency functions is very uncertain so that the larger the flood and the greater the hazard it represents, the less reliable is the estimate of the frequency with which the flood is likely to occur.

This uncertainty, mainly the fear of underestimating the potential magnitude of floods by using frequency functions, has raised the question of an upper limit that a flood wave from a given basin cannot exceed because of existing physical constraints such as watershed area, amount of water vapour in the air, etc. The search for this physically justified limit has led to the concepts of "maximum possible" or "maximum probable" flood. These concepts will be dealt with later since they belong to the category of flood synthesis based on analysis of climatic, meteorological, geographical, and other conditions rather than under flood analysis.

4.1.4 Problems in defining flood frequency

The result of flood frequency analysis is represented by a curve relating cumulative frequencies to the magnitudes of a relevant parameter, say, the flood discharge.
Usually, this curve tends to be automatically interpreted as a probability distribution function thus giving the probabilities of exceedance (cumulative frequencies) of the parameter. However, the statement "The flood with peak discharge $Q$, has probability of exceedance 5 per cent" can have two different meanings according to the method by which the frequency curve has been arrived at. It can mean:

(a) A flood which is exceeded, on the average, five times in 100 years; or

(b) A flood which is exceeded, on the average, in five years out of every 100.

The second statement says that in fact there can be more than five floods with peaks exceeding $Q$, but that they are confined to five years. This is the same as saying that in each of five individual years the maximum flood was higher than $Q$.

Both interpretations are being used to define the so-called "return period" (or recurrence interval) and the term "N-year flood", derived from it, which means a flood that is exceeded, on the average, (a) once in N years or (b) in one of N years.

The first interpretation is closer to the intuitive meaning of the N-year flood and it is based on the so-called partial-duration series of floods.

The second, on the other hand, can be more objectively assessed being based on maximum annual flows.

Related to the time unit of one year and to the notion of the N-year flood, the "probability" in case (a) is in fact not a probability but an annual frequency, since its value can exceed unity. This is a result of the construction of the partial-duration series which is described as follows.

From a T-year period of flow records a number of floods, $n$, is arranged in descending order with respect to a selected parameter. The number $n$ is arbitrary but usually greater than $T$. It depends on the discharge level above which the floods are counted which is also arbitrary and depends on whether, for the given purpose, one is interested mostly in large floods (as in spillway design) or small floods (as in the case of assessing agricultural losses in an alluvial flood plain). It is obvious that the ratio $P^* = \frac{m}{n}$, where $m$ is the order number of the variate in the descending series, does not define probability but the average annual frequency which is greater than unity for $m > T$. The N-year flood is given by $N = \frac{1}{P^*} = \frac{1}{m}$ so that the largest flood ($m=1$) is the T-year flood, the T-th one is the one-year flood, and for $m > T$ we get floods that occur more often than once a year. Thus we can have a half-year flood, a four-month flood, etc.

In case (b), which is gaining still more support among hydrologists, the frequency $P = \frac{m}{n}$ represents an estimate of probability since $n$ now is equal to $T$ and $P$ cannot exceed unity. In this case the shortest return period $N = \frac{1}{P}$ is one year (for $m=n=T$). Since the distribution is usually interpreted as continuous (the probability co-ordinates of the variates are computed from some plotting formula of the kind $P = \frac{m}{n}$ so that $P$ is always greater than unity) the return period of one year represents a limiting case which is approached asymptotically.
The relationship between $P$ and $P^*$ has the form

$$P = 1 - e^{-P^*}$$

(39)

based on the assumption that the events (say, floods with given peaks) are randomly distributed within given intervals (years). This implies that there can be intervals containing a number of events as well as intervals containing none. $P$ is then the probability that at least one event will occur in one interval.

Table VIII gives some frequently needed values of $P^*$, $N$ and $P$.

It can be seen, for instance, that the probability that a half-year flood will be exceeded in a particular year is $P = 0.865$. Now if the common interpretation of probability is used, we would expect this flood to be exceeded in about 87 out of every 100 years thus obtaining a return period $N = \frac{1}{0.865} = 1.157$ years and ending up with a paradox that the half-year flood is in fact a 1.157-year flood.

Fortunately, this problem arises only with small floods which are usually not the most important ones. As seen from Table VIII the values of $P$ and $P^*$ are closer to each other the longer the return period. In practical applications the above dilemma vanishes for return periods longer than about ten years which are approximately the same whether computed as $N = \frac{1}{P}$ or $N = \frac{1}{P^*}$.

Although the notion of return period based on annual frequency $P^*$ coincides better with the intuitive meaning of the term, there is one considerable disadvantage in dealing with the partial-duration series. The problem in question is the ambiguity of the notion of flood wave and following from it impossibility of finding a clear criterion for distinguishing one multi-peak flood from a number of floods in close succession. This becomes the more difficult when the lower discharge level is used for counting the floods.

The method based on annual maxima is more objective since it is independent of any discharge level and does not involve subjective decisions regarding multi-peak floods. The fact that for longer return periods it gives practically the same results as the partial-duration series further supports its application.

4.1.5 Problems in estimating flood frequency

To be able to estimate frequencies of flood parameters, especially for frequencies beyond the range of the period of records, it is necessary to postulate some theoretical model for the frequency distribution of the parameter concerned.

For this purpose the annual maxima rather than all maxima of the parameter are usually used, and the probability distribution function is sought rather than the "partial duration series". If the empirical sample contains all maxima, the corresponding probabilities of annual maxima can be found from equation (39). The empirical distribution function of annual maxima can then be plotted and a theoretical model fitted to it.
HYDROLOGICAL PROBLEMS ARISING FROM WATER EXCESS

TABLE VIII

Probabilities of exceedance, \( P \), and annual frequencies, \( P' \), of some
N-year floods

<table>
<thead>
<tr>
<th>( N ) (years)</th>
<th>( P )</th>
<th>( P' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>0.0010</td>
<td>0.001</td>
</tr>
<tr>
<td>100</td>
<td>0.00995</td>
<td>0.01</td>
</tr>
<tr>
<td>50</td>
<td>0.0198</td>
<td>0.02</td>
</tr>
<tr>
<td>25</td>
<td>0.0392</td>
<td>0.04</td>
</tr>
<tr>
<td>20</td>
<td>0.0488</td>
<td>0.05</td>
</tr>
<tr>
<td>10</td>
<td>0.0952</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>0.181</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>0.393</td>
<td>0.50</td>
</tr>
<tr>
<td>1</td>
<td>0.632</td>
<td>1.00</td>
</tr>
<tr>
<td>1/2</td>
<td>0.865</td>
<td>2.00</td>
</tr>
<tr>
<td>1/4</td>
<td>0.982</td>
<td>4.00</td>
</tr>
<tr>
<td>1/5</td>
<td>0.993</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Two main problems arise in connexion with the distribution model of a flood parameter:

(a) What type of model should be used;

(b) How best to estimate its parameters.

In general the problems are identical with those encountered in fitting distribution models to series of mean annual flows discussed in section 2. However, there is a slight difference in the present case because far greater emphasis is put on correct representation of the upper tail of the distribution which has a decisive role in estimating the magnitudes of floods with long return periods.
This concern has led to great efforts in finding distribution models which do not merely fit the empirical data well but which intrinsically reflect the fact that the variable (be it flood discharge, stage, or volume) represents the maximum of a certain number of homogeneous variates. Such distributions are known as distributions of extreme values and have been introduced in flood analysis mainly by Gumbel (for detailed account see (45)).

The problem with the distributions of extremes lies in the fact that their theory has so far been developed only for extremes from mutually independent events which assumption is almost never satisfied in the case of flood parameters. For instance, if the maximum annual flood discharge is characterized by the maximum annual mean daily flow (as is the case with most larger rivers), then the annual maxima would be distributed as extremes from sets of 365 daily flows if the latter were mutually independent which is far from true.

Nevertheless, the distributions of extremes are being generally used for fitting to flood parameters. The three most frequently used models are the extreme or Fisher-Tippett, types I, II, and III, type I being also known as the Gumbel distribution. A detailed account of applications of these models to distributions of flood parameters is given by Jenkinson in (7).

Some authors argue that, the basic assumption of independence not being satisfied in the parent sample of flood parameters, there is no advantage in using extremal distributions for fitting to samples of their annual maxima. Moran ((36), page 95) argues that another weakness in using the extremal distribution resides in the fact that it depends on the upper tail of the parent distribution whose correct shape itself can be only guessed at and can be hardly considered a solid basis for estimating the parameters of the extremal distribution. For practical purposes he recommends the use of simple distributions such as the gamma or the log-normal types whose two parameters can be reasonably well estimated directly from the sample of the annual maxima. Practical applications of these two distributions are also described in reference (7).

In addition to the uncertainty in the distribution model, there is a considerable uncertainty in the estimates of model parameters. Although the method of maximum likelihood has been generally accepted as being the best for such estimates, it is not simple to use and less efficient but more convenient methods have been suggested as described in section 2 and in detail in reference (7).

There is one more problem that has to be considered in modelling the flood frequency distribution. In many rivers floods within one year cannot be regarded as homogeneous events. Typically, they may be caused by two or more distinctly different physical situations. For instance, floods caused by snowmelt will have different patterns than those caused by summer convective storms, and these may differ considerably from floods caused, say, by hurricanes. Often the result of this non-homogeneity of annual maxima is an empirical distribution function with sharp jumps or other irregularities preventing it from being satisfactorily fitted with any usual model. The usual but incorrect practice is to use different models for individual fragments of the empirical distribution function. This purely formalistic approach has neither statistical nor physical justification and cannot be recommended. In
the case of the described non-homogeneity the distribution of annual maxima is a mixture of distributions of all the homogeneous components and should be treated accordingly (46).

According to the distribution model chosen and method of parameter estimate used, the shape of the fitted distribution function will vary and with it the error in extrapolated flood frequencies. It may well happen that the same flood discharge will correspond to a 1 000-year return period according to one method of fitting and to a 10 000-year period according to another while both fits can be statistically acceptable.

This uncertainty is inherent to flood frequency analysis and cannot be removed by any mathematical tricks. The decision as to what result to use for design purposes rests with the designer and depends entirely on his judgment and extra-hydrological features of the particular project.

The situation is simplified in some countries (e.g. the U.S.S.R.) by standardization which prescribes the type of distribution model and sometimes even the method of parameter estimates to be used in flood frequency analysis.

Another form of circumventing these difficulties is to refrain from estimating a return period of large floods and instead try to find a physical limit that cannot be exceeded (see section 4.1.6).

Another problem arises in using a distribution model for assessing the chance of flood occurrence within a given period of time. Its basis is a misinterpretation of the notion of "N-year flood", widespread among practising engineers, consisting in the wrong belief that, say, a 100-year flood is not likely to occur much sooner than 100 years from its last occurrence, in other words, that there will be no 100-year flood for a period of about 100 years. The fact is that the chance that this will happen is rather remote and can be assessed on the assumption that occurrences of floods in individual years are independent. If this assumption is valid then the probability that an N-year flood will not occur during T successive years, which can be called degree of safety, is given by

$$s = (1 - \frac{1}{N})^T$$

(40)

The probability, or the risk, that such a flood will occur at least once, is then

$$r = 1 - s.$$  

(41)

For some values of N and T the degrees of safety computed by equation (40) are shown in Table IX.

The awareness of the actual degree of safety associated with a given return period is of great importance in design. For instance, if a coffer-dam protecting a construction site of a hydro-power plant should be almost absolutely safe (with, say, degree of safety $s = 0.99$) for a period of ten years within which the plant should be completed, then it must be designed to withstand an approximately 1 000-year flood since from equation (40) we have $0.99 = (1 - \frac{1}{N})^{10}$ which yields $N = 996$ (see also Table IX).
TABLE IX

Probability that an N-year flood will not occur or be exceeded during a period of T years (degree of safety against N-year flood in T years)

<table>
<thead>
<tr>
<th>N</th>
<th>10</th>
<th>20</th>
<th>50</th>
<th>100</th>
<th>1000</th>
<th>10000</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.3487</td>
<td>0.1216</td>
<td>0.5154 $10^{-2}$</td>
<td>0.2656 $10^{-4}$</td>
<td>0.1748 $10^{-5}$</td>
<td>0.2661 $10^{-6}$</td>
</tr>
<tr>
<td>20</td>
<td>0.5987</td>
<td>0.3585</td>
<td>0.7694 $10^{-1}$</td>
<td>0.5921 $10^{-2}$</td>
<td>0.5292 $10^{-2}$</td>
<td>0.1722 $10^{-2}$</td>
</tr>
<tr>
<td>50</td>
<td>0.8171</td>
<td>0.6676</td>
<td>0.3642</td>
<td>0.1326</td>
<td>0.1683 $10^{-8}$</td>
<td>0.1823 $10^{-8}$</td>
</tr>
<tr>
<td>100</td>
<td>0.90438</td>
<td>0.8179</td>
<td>0.6050</td>
<td>0.3660</td>
<td>0.4317 $10^{-8}$</td>
<td>0.2249 $10^{-8}$</td>
</tr>
<tr>
<td>1000</td>
<td>0.990045</td>
<td>0.98019</td>
<td>0.95121</td>
<td>0.90479</td>
<td>0.3677</td>
<td>0.4517 $10^{-5}$</td>
</tr>
<tr>
<td>10000</td>
<td>0.9990004</td>
<td>0.996002</td>
<td>0.995012</td>
<td>0.99049</td>
<td>0.90483</td>
<td>0.3678</td>
</tr>
</tbody>
</table>

\[ P = (1 - \frac{1}{N})^T \]
4.2 FLOOD SYNTHESIS

Flood synthesis is usually employed in connexion with the estimation of the so-called design flood.

Design flood is a flood hydrograph used as the basis for the design of engineering structures or for making water-management decisions. In general, the design flood is the maximum flood by which the given structure or decision should not be adversely affected. Hydrologically speaking, the design flood can either be a flood of a specified return period (N-year flood), or a so-called maximum probable flood. Sometimes a recorded historical flood is used as the design flood.

From the point of view of practical water management the current approach to flood analysis has one considerable weakness, namely that a flood is not being analysed as a coherent physical entity but is first split up into a number of components which are then analysed separately. As a result one can get a reasonable idea about, say, a 100-year flood flow, flood volume, flood duration, etc., but no clear idea about the hydrograph of a 100-year flood. In fact, an N-year flood can hardly be satisfactorily defined since it is very rare for a real flood that all its parameters have the same return period. The N-year flood is an artificial creation, an idealized concept intended to simplify flood management and the design of structures affected by floods.

Being non-existent in nature, the N-year flood has to be synthesized from the raw results of flood analysis or, in some cases, of precipitation analysis.

Since the most evident and often the most important flood parameter is its peak discharge, the most frequent concept of the N-year flood is a flood whose peak discharge has the N-year return period and which otherwise exhibits either a "typical" behaviour (average volume, duration, etc.) or the most unfavourable behaviour (largest volume, steepest rise, etc.), or it eventually can mean the N-year discharge itself. The last case can be interpreted as a flood whose peak discharge is equal to $Q_N$ and lasts for a long, but unspecified, period of time.

If data on recorded floods are scarce or unreliable, it is often the case that before the flood hydrograph can be synthesized, the peak discharge itself must first be found by using synthesis, or as it is also called, indirect methods.

The second artificial concept is the so-called maximum probable flood which should represent the physical limit that is not likely to be exceeded in given climatic and geographic conditions.

4.2.1 Indirect determination of flood flows

4.2.1.1 Rational method

The rational method is often used for estimating flood flows from small drainage systems, mainly in urban areas and from small watersheds where data on floods are not available and direct flood flow analysis is not possible. The method
has been widely used in engineering practice, mainly for the estimation of design discharge rates for culverts, drainage outlets, sewers, small bridge-openings, etc.

The basic formula gives discharge as

$$Q = C I A$$ \hspace{1cm} (42)

where $A$ is drainage area, $C$ is a coefficient called the "runoff coefficient", and $I$ is intensity of rainfall of a duration equal to "time of concentration" which means a period equal to that required for the full area $A$ to contribute to outflow.

The main drawbacks of the rational method are the following:

(a) The return period of $Q$ is not the same as that of $I$, as is usually assumed in practical applications. In general, the same rainfall can give rise to very different runoff rates depending on the antecedent conditions of soil moisture which are difficult to predict accurately;

(b) It is extremely difficult to estimate the correct value of $C$ not only because of the usual topographical non-homogeneity of the drainage system concerned, but also because it depends on soil moisture conditions and to some extent on rainfall intensity;

(c) The estimate of the time of concentration presents another difficulty because it also depends on the antecedent conditions in the basin.

By and large, the rational method gives reasonable results only if applied by an experienced expert, or if there are enough experimental data to remove the above uncertainties. In the latter case, however, there would be no need for the rational method because there are other more reliable methods that can be used in the presence of data.

4.2.1.2 Regional empirical formulae

It is known from experience that specific runoff for flood peaks, $q_{\text{max}}$, decreases with the increase of drainage area $A$ which can be explained from the fact that the larger the basin, the longer, in general, is the time of concentration and hence the lower the intensity of rainfall of a given return period. The relationship can usually be well approximated by a power function.

$$q_{\text{max}} = \frac{a}{A^b}$$ \hspace{1cm} (43)

where $a$ and $b$ are empirically found parameters. Since by definition $q_{\text{max}} = Q_{\text{max}}/A$, we have

$$Q_{\text{max}} = \frac{a}{A^{b-1}} = \frac{a}{A^c}$$ \hspace{1cm} (44)
The coefficients $a$ and $c$ corresponding to flood flow of a given return period $N$ can be found by plotting basin areas against respective values of $Q_N$ (found from frequency analysis of recorded floods) on double-log paper where equation (44) appears as a straight line since

$$\log Q_{\max} = \log a - c \log A.$$  \hspace{1cm} (45)

Formulae of the above type ((6), (8), (17)) have been used in the U.S.A. (Meyer, Bureau of Public Roads Method), the U.S.S.R. (Kocherin), Czechoslovakia (Dub), and other countries, and are usually set up for the 100-year discharge ($Q_{\max} = Q_{100}$) from which other $N$-year flows are derived by empirical multiplication constants.

In recent years there has been a tendency to abandon this type of formula, mainly because of its static nature which does not take into account "... the changing relationships between the numerous variables during the course of the runoff-producing storm" (Ward, (47)). However, the same objection could be directed towards the whole area of frequency analysis of which the above approach is only a regional generalization.

Regardless of the defects that the described method certainly has, it at least offers a consistent, and in principle physically justified, approach for estimating $N$-year flows in ungauged basins.

### 4.2.2 Synthesis of flood hydrographs

Synthesis of flood hydrographs has become one of the main objectives of applied hydrology for two different and largely independent reasons. The first is flood forecasting, the second is design of structures affected by floods.

In flood forecasting the main emphasis is on the dynamic side of the problem and the relevant methods are not discussed in this report.

Here, methods are described which are used in design-related synthesis where the main emphasis is on the statistical side of the problem, in general on the synthesis of hydrographs of floods with specified return periods.

The methods can be divided into two general categories, the first utilizing information from the analysis of historical floods alone, the second making use of information on historical floods as well as that on rainfall and other phenomena affecting the properties of floods.

The most important representatives of the second category are the methods of the unit hydrograph and digital simulation technique mentioned in section 2. These methods, although hydrologically superior to those of the first category are more appropriate for hydrological forecasting where there is no need of defining the return period of the synthetic flood. The difficulty in assessing return periods in these methods arises (a) from their dealing with rainfall intensities whose return periods are related to rain durations, (b) from the fact that it is hard to assess return periods of all components entering the synthesis and even if this could be done it would be impossible to determine a unique return period of the resulting hydrograph from those of its components.
These difficulties cannot be removed since they do not reflect technical
imperfections of the method but rather the intrinsic properties of flood formation.
They clearly demonstrate that there is nothing of the sort of, say, a unique 100-year
flood, and that, at the best, there is a distribution of 100-year floods.

For these reasons, the unit hydrograph and related methods are used for
design purposes only if the design flood is a priori related to a specified rainfall
duration (as it often is in design of sewers, culverts, etc.) or if it should be
close to a "probable maximum" whose return period is unspecified.

In the following, a brief account is given of some methods of the first
category which is more relevant to the notion of design flood; the second category
is represented by a short outline of the synthesis of the maximum probable flood.

4.2.2.1 Geometric methods

These methods (described in (2), (6), (7)) are intended for cases with
insufficient data on streamflow. The simplest version (Kocherin) approximates the
flood hydrograph by a triangle determined by the peak flow $Q_{\text{max}}$, corresponding flood
volume $V$, and the ratio $t_1:t_2$ between the durations of the rising ($t_1$) and falling
($t_2$) limb of the hydrograph.

The peak flow is obtained from frequency analysis or calculated by an
empirical regional formula, the volume is found on the basis of an estimated
coefficient of runoff and an observed or estimated flood-producing rain, the
duration of flood is given as $t = t_1 + t_2 = 2V/Q_{\text{max}}$, and the ratio $t_1:t_2$ is selected
according to the basin area as follows:

1:1 for basins between 300 and 500 km$^2$,
1:1.5 for basins between 500 and 5 000 km$^2$,
1:3 for basins over 5 000 km$^2$.

There are many refinements of this basic method consisting in representing
the hydrograph by a trapezium, segments of parabolas, etc. (see (6), (7)).

The synthetic hydrograph is usually interpreted as a 100- or 50-year flood
(depending on the nature of underlying data) and hydrographs for other return periods
are derived from the basic one via the estimation of $Q_{\text{N}}$ from the basic $Q_{\text{max}}$ (using
empirical or theoretical multiplication constants) and a proportional change of flood
volume.

Results obtained by these methods must be regarded as very rough estimates
and used with extreme caution.
4.2.2.2 Correlation methods

In cases where adequate data on historical floods are available, frequency analysis of flood elements and correlation analysis of their interrelationships can be used for synthesis of what can be called typical N-year floods useful mainly for economical optimization of flood protection schemes.

The general framework of these methods is such that the magnitude of the leading parameter, usually the peak discharge, is chosen by frequency analysis and all other parameters (volume, duration, various shape parameters) are found from their regression on the leading parameter. If no significant correlation for certain parameters is found, the average value is used.

4.2.2.3 Equal probability methods

These theoretically unjustified but frequently used methods synthesize the hydrograph from elements of equal return periods. This usually tends to overestimate the volume of floods with high peak flows since large flood volumes are usually produced by prolonged rains of relatively low intensities resulting in floods with flat peaks.

One version used as a standard in the U.S.S.R. employs peak discharge and volume of equal return periods while the model for hydrograph shape is chosen from hydrographs of recorded floods in such a way that the synthetic hydrograph represents the "most dangerous" variant within the given return period.

Another version described in (6) uses the envelopes of volume curves of recorded floods in the following way. The peak discharge $Q_N$ is found by frequency analysis and is plotted on the discharge axis of the chart showing volume curves of all analysed floods. For any $Q_i < Q_N$, $i = 1, 2, \ldots, m$ the preliminary volume is found as the abscissa having the same number of exceedances as $Q_N$. The final N-year volume curve is an envelope of all $m$ points representing volumes of equal exceedances (Figure 16). The actual shape of the synthetic hydrograph is found by using as a model a typical hydrograph derived from comparable historical floods.

4.2.2.4 Maximum probable flood

The concept of maximum probable flood has evolved from the experience that a flood was recorded on a river (e.g. Miami River, Ohio, 1913) that by far exceeds all other recorded floods, and its occurrence cannot be satisfactorily explained by conventional methods of frequency analysis. From this it has been inferred that such extraordinary floods can occur on any river and that frequency analysis does not provide a reliable enough picture of actual flood danger. Although a frequency curve can be extrapolated ad infinitum and any magnitude of peak flow can be assigned some return period, this exercise seems meaningless after we pass a certain limit beyond which intuition fails as a guide for what is and is not reasonable. If, for instance, a recorded outlier on river A indicated a return period of 10 000 years and one on river B a period of, say, a million years, it may be inferred from the frequency function that river A can also experience a million-year flood which would be many
Figure 16 - Definition sketch for equal-probability volume curve of flood
times larger than the recorded one. However, there may be such physical conditions on river A which would make the occurrence of such a flood virtually impossible. The purpose of the maximum probable flood is to assess to what extent the extrapolation of maximum flows can be considered reasonable and physically justified.

The procedure is described elsewhere, (for a very detailed account see (7)) and here it will be only briefly summarized: For the basin in question a broader region is identified within which the climatic conditions can be regarded as homogeneous and where it can be assumed that any observed rain-producing storm could have occurred at any location within the region. All storms observed in the region are then transposed over the analysed basin and if there are no reasons for the contrary (say, orographical obstacles) orientation of their isohyetal pattern is changed to produce the maximum possible rainfall on the basin territory. Another adjustment that is made is the maximization of air moisture content. The recorded amount of rainfall is then increased by the ratio of the precipitable amount of water to the amount actually precipitated. The resulting rainfall is finally converted into runoff by the method of the unit hydrograph.

### 4.2.3 Flood routeing

The term flood routeing is used for methods for approximate solution of unsteady flow of water in river channels and through storage reservoirs. Exact solutions of these problems lead to non-linear differential equations which have no simple closed solutions even for the most regular geometry of channels and reservoirs.

For most practical purposes the difficult and laborious exact solutions are not justified because of the uncertainties inherent in the data, and the routeing methods have been found adequate.

#### 4.2.3.1 Reservoir routeing

Reservoir routeing is concerned with the transformation of a flood wave during its passage through a storage reservoir. In other words, the objective is to find the shape of the flood hydrograph down-stream of a dam given the hydrograph up-stream.

The reservoir is characterized by its volume curve, \( S = f(H) \), relating reservoir storage and elevation of water table, and by combined rating curve of all outlets, \( Q' = g(H) \), relating reservoir outflow and elevation of water table.

The term reservoir routeing implies certain simplifying assumptions which are approximately satisfied in relatively deep reservoirs but are not satisfied in river channels and long-and-shallow reservoirs. These conditions are:

(a) Water surface in reservoir is approximately horizontal under any conditions of inflow and outflow;
(b) Response of output to any reservoir input is almost immediate (the response is mainly due to transfer of hydrostatic pressure rather than due to transfer of water mass as it is in long river reaches).

Under these assumptions the theoretical basis of routeing methods is represented by the continuity equation in a simple difference form

\[ Q \Delta t - Q' \Delta t = \Delta S \]  

which can also be written as

\[ (Q-Q') \Delta t = S_{i+1} - S_i \]  

where \( Q \) is the rate of input, \( Q' \) is the rate of output, \( \Delta t \) is a time interval, and \( \Delta S \) is the difference in reservoir storage between the beginning of the interval \( S_i \) and its end \( S_{i+1} \).

The routeing procedure consists in solving equation (47) for output \( Q' \), for successive discrete time instants \( \Delta t \) apart. This must be done by successive approximation since the rate of output at the end of the interval depends on \( S_{i+1} \) which, however, is not known at the beginning of the interval. The iterative method presents no problem when the solution can be carried out on a high-speed digital computer.

There are many direct graphical solutions (summarized in (48)) which provide quick and sufficiently accurate results and in addition offer a good insight into the problem.

One of the simplest graphical routeing methods is due to Klemeš (49) and consists in the following. The "routeing curve" giving the relationship between reservoir storage and rate of outflow \( Q' = f(S)_r \) (compiled from reservoir volume curve and rating curve of outlets) and the input hydrograph \( Q = g(t) \) are plotted abreast so that the discharge axes have the same orientation and scale and the zero point of the flood-control storage on the routeing curve corresponds to the detention limit of discharge, \( Q_k \) (the flood-control storage remains empty until input reaches the value \( Q_k \); lower flows are being released through turbines or bottom outlets of the dam). In the plot the scales of time and storage are arbitrary as well as the length of the time interval \( \Delta t \). In the graph of the routeing curve the angle \( \delta \) is determined for a "reduction line" connecting some arbitrary value of output, say \( Q' \), on the output axis with the value \( S_x = \Delta t Q' \) on the storage axis. The purpose of the reduction line is to convert graphically the discharge into the volume of water produced by this discharge during the interval \( \Delta t \) (Figure 17).

The input hydrograph is split up into intervals \( \Delta t \) of which only the centre lines are plotted. Their intersections with the hydrograph define points \( Q_0, Q_1, \ldots, Q_i, \ldots \) and the objective is to find on the centre lines the corresponding points of the output hydrograph \( Q_0', Q_1', \ldots, Q_i', \ldots \).
Figure 17 - Klemeš flood-routing method
The procedure of finding the point $Q_{i+1}$ is as follows (Figure 17). We are at the $i$-th time interval thus knowing the values $Q_i, S_i$, and, of course, all the points $Q_{i+1}, Q_{i+2}, \ldots$ A horizontal line is drawn through point $Q_{i+1}$ and a vertical line through point $S_i$, their intersection is point $A$. A straight line is drawn through point $A$ under angle $\delta$ and its intersection with the routeing curve defines point $B$. The abscissa of $B$ is equal to $S_{i+1}$ and therefore its ordinate represents the output at time $\Delta t_{i+1}$. By drawing a horizontal line through $B$, point $Q_{i+1}$ is obtained on the $(i+1)$-th centre line.

Apart from the output hydrograph we obtain the total detention storage needed during the passage of the flood through the reservoir; it is given as $S_{\text{max}}$ in the routeing curve chart. The method can easily accommodate variable time intervals that can be used for improving the accuracy around the top of the output hydrograph.

Figure 17 demonstrates the basic procedure for reservoir outlets with fixed openings (e.g. an un gated spillway) or for a case when the gates remain in a fixed position during the flood passage. However, the above method can also be used in cases where the gates are being operated during flood passage (49). This is a very frequent case in dam design, for instance, when operating rules for the gates are to be found, leading to a prescribed shape of the outflow hydrograph for a given shape of the input hydrograph (usually represented by a design flood).

Another example of graphical routeing procedures is the Puls method, known as Potapov's method in Russian hydrological literature. It is often quoted in hydrological textbooks and is based on equation (47) written in the form

$$\frac{1}{2} (Q_i + Q_{i+1}) \Delta t + \frac{1}{2} (Q_i + Q_{i+1}) \Delta t = S_{i+1} - S_i \quad (48)$$

where subscripts $i$ and $i+1$ relate to the beginning and the end of interval $\Delta t$, respectively. This method uses two routeing curves, $Q' = f_1(S + 0.5Q_i \Delta t)$, $Q' = f_2(S - 0.5Q_i \Delta t)$, and is convenient for work with constant time intervals and fixed reservoir outlets.

### 4.2.3.2 Channel routeing

For the movement of a flood wave along a channel, the assumptions used in reservoir routeing do not hold and other methods must be used for determination of the transformed shape of the flood hydrograph.

One difference is that while in an ideal reservoir storage can be related to the rate of outflow alone (water table is considered horizontal and thus can be measured at the outlet where it also uniquely defines the rate of outflow), in a channel reach it depends both on outflow and inflow because of the relatively significant influence of water table slope.

Another difference is that the travel time through a river reach cannot be neglected as has been possible with the travel time through the ideal reservoir.
A well-known method for the channel-routing problem is the Muskingum method by McCarthy (see, for instance, (8)) where the two above properties of a river reach are expressed by empirical formula

$$S = K\left[xQ + (1-x)Q'\right].$$  \hspace{3cm} (49)

Here $S$ is storage in a river reach, $Q$ is inflow, $Q'$ is outflow, $x$ is a weighting factor taking on values between 0.5 and 0, and $K$ is a constant approximately equal to the travel time through the reach.

Both $x$ and $K$ are found empirically from hydrographs of recorded floods in the following manner. The volumes in the river reach, $S_i$, at instants $t_i$, $i = 0, 1, \ldots, n$, represented by the area between inflow and outflow hydrographs (Figure 18), are plotted against the values $xQ_i + (1-x)Q'_i$, for a set of arbitrarily chosen values of $x$, say $x = 0, 0.1, 0.2, \text{ etc.}$ Each plot will form a loop whose two branches will collapse into one line (Figure 18 (b)) for a certain value of $x$ which is then regarded as the correct one. The value of $K$ is given by the slope of the line having the correct value of $x$.

Using the values of $K$ and $x$, the outflow hydrograph can be found for any inflow by means of a step-by-step procedure in which output $Q'_i$ at time $t_i$ is given as

$$Q'_i = c_0Q_i + c_1Q_{i-1} + c_2Q'_{i-1}$$ \hspace{3cm} (50)

the constants $c_0$, $c_1$, $c_2$ being the following functions of $x$ and $K$:

$$c_0 = -\frac{Kx - 0.5\Delta t}{K-Kx + 0.5\Delta t}$$

$$c_1 = \frac{Kx + 0.5\Delta t}{K-Kx + 0.5\Delta t}$$ \hspace{3cm} (51)

$$c_2 = \frac{K-Kx - 0.5\Delta t}{K-Kx + 0.5\Delta t}$$

For floods with short times of concentration (steep rising limb) the Muskingum method tends to give negative starting values and it is then advisable to use some other method, for instance the unit reach method by Kalinin and Milyukov (50). It represents river reach by $n$ equal linear reservoirs (ideal reservoir with routing curve represented by a straight line) and assumes travel time through the reach equal to $\tau$.

The outflow hydrograph is then given as

$$Q'(:t) = Q \frac{\Delta t}{\tau^{n-1}} \frac{t^{n-1}}{\tau^{n(n-1)}} e^{-t/\tau}$$ \hspace{3cm} (52)

where $Q$ is the mean inflow during period $\Delta t$. 
Figure 18 - Definition sketch for Muskingum method
As shown by Svoboda (51) the constants for the unit reach method can be found from those of the Muskingum method as follows

\[ n = \frac{1}{1-2x}, \quad (53) \]

\[ \tau = K/n. \quad (54) \]

This makes the unit reach method a convenient substitute for the Muskingum method in cases where the latter is not suitable. The two methods are complementary in the sense that the Muskingum method gives good results with long river reaches while the unit reach method is more suitable for short reaches.

### 4.2.4 Flood control

The purpose of flood control is to eliminate or to reduce damage caused by the flooding of areas adjacent to rivers or by overtopping engineering structures like dams, embankments or bridges. Two basic engineering means of flood control are:

(a) Reservoirs where part of the flood waters can be temporarily retained, and thus the flood peak reduced and the passage of water masses stretched over a longer period of time at a reduced flow rate;

(b) Levees built along rivers to protect the adjacent territory by confining the flood to a particular channel; in this case the flood peak may be increased because of the elimination of storage of the former inundation area.

It has to be borne in mind that in both cases the original travel time of a flood will be changed and this may have serious consequences in down-stream parts of the river. It may well happen that, for instance, by reducing flood peaks on a tributary, flood peaks on the main river down-stream from the confluence are increased due to a change in the time which will cause the floods on the tributary and the main stream to coincide.

The hydrological basis for flood control is the so-called design flood (see section 4.2) for which the efficiency of intended flood protection is tested. To decide what flood will be used as the design flood is beyond the scope of hydrology. The decision depends on economic, social, and political factors. There are, however, two general rules that are observed with few exceptions:

1. If flood protection is required for economic reasons only, then the design flood is chosen by optimizing the balance of costs and benefits. As a result, the design flood may often be one with a comparatively short return period. This is typical in flood protection of agricultural land;

2. If human life is endangered, then an "absolute" protection is usually required which practically means that the maximum probable flood (see paragraph 4.2.2.4), or the maximum recorded
flood*, is employed as the design flood. This is typical in cases such as spillways on earthen dams where overtopping of the dam would cause its destruction, levees protecting residential districts, etc.

From the methodological point of view, flood control by means of levees involves only flood routing techniques, discussed in the preceding section, and it will not be dealt with here. The only new aspect is the reversed order of the solution. (By eliminating inundation we eliminate storage and the original flood is now represented by the outflow hydrograph while the result of solution is the inflow hydrograph, i.e., that with the storage effect removed.)

The methodology of flood control by means of storage reservoirs cannot be reduced to routing techniques and will be covered in more detail.

4.2.4.1 Controlled and uncontrolled reservoir storage

The current understanding is that controlled and uncontrolled storage is identical with gated and ungated storage, respectively. This, however, is grossly misleading and could lead to dangerous consequences in reservoir flood-control.

It is true that a fully controlled storage must have a gated outlet but a gated outlet does not automatically guarantee full control over the storage. In order that the storage be fully controlled the outlet capacity corresponding to zero storage must be equal to or greater than the maximum possible rate of inflow. If this condition is not met the filling of storage at high inflows cannot be prevented and control over the rate of the filling is limited once the inflow exceeds outlet capacity. Thus the degree of storage controllability depends on the capacity of outlets which, of course, must be gated.

On the other hand, in most dams the storage above the spillway crest is not entirely uncontrolled even if the spillway has no gates. The reason for this is the fact that, as a rule, the dam has a gated bottom outlet and that by operating its gates the filling of storage above the spillway crest can be partly controlled. The degree of controllability again depends on the capacity of the gated bottom outlet. However, here the control can never be complete because even with the outlet closed the emptying of the storage above an ungated spillway crest cannot be prevented.

Controlled detention (flood-control) storage is the most efficient means for reducing flood peaks for in this case the reduction to certain flow $Q_k$ (a non-damaging flow) requires the minimum storage capacity which is equal to the volume of the flood above the discharge $Q_k$. In Figure 19 this volume is denoted by $A$ and the corresponding ideal output hydrograph is $Q_A^t$. In this case the detention storage can be emptied (and prepared for the next flood) within a time $t_A$ after inflow has dropped below $Q_k$ which again represents the shortest time possible (the actual emptying time is slightly longer since the reduction of outflow from $Q_A^t$ to $Q$ is never instantaneous).

* In southern Ontario, for instance, floods caused by the hurricane "Hazel" (1956) are often used as design floods.
If the same reduction of flood peak were to be achieved by an uncontrolled storage the required storage would be $A + B$ (Figure 19). The portion $B$ is hydrologically ineffective since it is wasted (a) on reduction of flows that do not have to be reduced, period $t_0 + k$; (b) on unnecessarily high reduction of flows, period $t_1$. Naturally, the time needed for emptying the storage, $t_{A+B}$, is much longer than $t_A$.

The first necessary condition for the effectiveness of a controlled storage is, of course, a sufficient capacity of gated outlets which must be at least $Q_A$. If it were lower, say $Q_{<}$, the whole detention storage would be filled even before the input reaches the damaging flow $Q_{k_1}$ (time $t_1$ in Figure 19) and there would be no storage left to reduce the peak. Theoretically, from time $t_1$ on the outflow hydrograph would be identical with that of inflow. What would happen in practice is this: the reservoir would keep on filling until the water table would reach the dam crest at some time $t_2$; then the whole dam crest would start functioning as an ungated spillway (see hydrograph $Q'(A)$ in Figure 19) and the overtopping could cause destruction of the dam.

The second condition is that the gates be operated in such a way as to prevent any filling of detention storage before the input reaches $Q_s$. To meet this condition requires high responsibility and physical endurance from operating personnel (permanent duty during flood danger), as well as highly reliable gate mechanisms and power supply.

Generally speaking, the higher efficiency of controlled storage as compared to uncontrolled storage is always reached at the expense of the overall safety. The usual arrangement on dams is therefore such that (1) at least an "emergency" ungated spillway is provided with its crest not higher than the elevation of the normal maximum flood level in the controlled detention storage, (2) the freeboard (vertical distance between the normal maximum flood level and the dam crest) is at least one metre more than necessary for passing the total design flood over the emergency spillway.

4.2.4.2 Design of a flood-control reservoir

The main conceptual difference between the design of a reservoir for low-flow regulation and for flood-control resides in the fact that in the former case one takes into account the periods between successive water shortages and uses as a basis the whole time series of streamflows, while in the latter case the periods between successive floods are considered long enough to release the stored water and storage capacity is determined on the basis of one single flood.

The reason for not taking into account the periods between floods is the assumption that the time needed for emptying the detention storage is always much shorter than the shortest expected period between two floods of the magnitude comparable to the design flood. Although this is usually the case, there are certain climatic conditions where the occurrence of two major storms in close succession can reasonably be expected (usually areas affected by tropical cyclones like typhoons or hurricanes). In such cases the period between floods cannot be ignored.
Storage capacity of a reservoir with uncontrolled detention storage is designed by a reservoir-routing method. Knowing the maximum desired flow downstream $Q_k$, we route the design flood through the reservoir assuming a certain size and shape of outlets, and compare the peak discharge of the output hydrograph, $Q'_\text{max}$, with $Q_k$. We then keep changing the size and shape of outlets until $Q'_\text{max} = Q_k$. The required storage capacity is given by the value of $S_{\text{max}}$ corresponding to $Q'_\text{max}$ in the plot of the routing curve (Figure 17).

The capacity of controlled detention storage could in theory be easily found by making it equal to the volume of the design flood above the discharge $Q_k$. This simple procedure has, however, one defect consisting of the fact that the return period of the design flood relates as a rule to its peak flow, not necessarily to its volume. On the other hand, the detention storage capacity depends here exclusively on flood volume, the peak discharge being irrelevant. For this reason, if N-year protection against a certain discharge $Q_k$ is required, the volume of the N-year design flood does not give the correct answer unless the N-year flood has been set up on the basis of volumes rather than peak flows as is usually the case.
A recommended procedure for design of controlled detention-storage capacity required for N-year protection against discharge $Q_k$ is as follows (Figure 20).

(a) Volume curves of floods are plotted separately for every year of records and envelopes of these annual sets of curves are drawn. These annual envelopes are plotted into one chart (Figure 20 (a));

(b) For a set of discharge values $Q_1$, $Q_2$, ..., $Q_n$, ..., frequency curves of volumes $A_1$, $A_2$, ..., $A_n$, ..., are plotted and fitted with theoretical distribution functions to facilitate extrapolation of the upper tails. It is to be noted that the number of empirical points on the volume frequency curve for each discharge level is the same, and is equal to the number of years, $n$ (i.e. number of curves in Figure 20 (a)). However, as the discharge rate increases, still more of the $n$ values of volume reduce to zero; these zero values must be retained in each frequency curve (Figure 20 (b)).

(c) Frequency curves of flood volumes are converted into return-period curves $B_1$, $B_2$, ..., $B_n$, ..., using the return period $N = 100$ per cent as the abscissa (Figure 20(c));

(d) Vertical cross-section of the family of $B$-curves at $N$ defines the relationship between discharge $Q$ and flood volume $S$ for return period of $N$ years. These relationships are plotted for several typical values of $N$ (say, 50, 100, 1 000 years) forming a family of curves $C_N$ (Figure 20 (d)).

From the family of $C_N$ curves the correct value of controlled storage can be found for N-year protection against arbitrary discharge $Q_k$ (Figure 20 (d)).

Figure 20 (d) demonstrates one often encountered fact, namely that the storage needed for a relatively high protection is usually not much larger than storage for comparatively small protection. In the example shown, storage capacity $S_{1000}$ required for 1 000-year protection against $Q_k$ is only about 20 per cent larger than that needed for 100-year protection against the same discharge. This is important to be aware of in design practice where as a result of optimization the optimum degree of protection is often relatively low. Although there is no reason to increase the degree of protection unless it is well justified, the designer should also consider the limited accuracy of economic data on which the optimization is based and weigh this against the cost of additional protection, especially if the computed optimum charge is not sharp.

4.2.4.3 Flood control by a multi-purpose reservoir

In almost every multi-purpose reservoir one of the functions is flood control and detention storage is provided for this purpose. This storage is designed in the same manner as described in the preceding paragraph.
Figure 20 - Definition sketch for design of storage capacity required for N-year protection against discharge $Q_k$. 
However, a multi-purpose reservoir with a given detention storage capacity can render more efficient flood control than a purely flood-control reservoir of the same detention storage. The increase of efficiency is roughly proportional to the conservation storage of the multi-purpose reservoir whose flood controlling effect is due to the fact that conservation storage is usually not full at the beginning of a flood, and the empty portion of it can be used as an ad hoc supplement to detention storage. The availability of this additional storage does not have to be left to chance (the part of zone 5 which lies within storage S in Figure 9) and usually it is embedded in reservoir operating rules. Its magnitude at any particular time of year depends on seasonal fluctuations of streamflow and is found by statistical analysis thereof.

A quantitative evaluation of the additional effect of conservation storage on flood control can best be done by the Monte Carlo simulation of reservoir operation. In current design practice, however, this effect is often not being evaluated and is regarded as an increased safety margin.

Flood control by a multi-purpose reservoir is far too complex a problem to be described adequately in a short paragraph, the complexities being both of a hydrological and water-management nature. For a deeper insight into the problem the reader is referred to a comprehensive paper on the subject by Beard (52).
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