TECHNICAL NOTE No. 58

TIDAL PHENOMENA IN THE UPPER ATMOSPHERE

B. HAURWITZ

WMO-No. 146. TP. 69

Secretariat of the World Meteorological Organization – Geneva – Switzerland
THE WMO

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— to promote standardization of meteorological observations and ensure the uniform publication of observations and statistics,
— to further the application of meteorology to aviation, shipping, water problems, agriculture, and other human activities,
— to encourage research and training in meteorology.

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FOREWORD

At its third session (Rome, 1961), the WMO Commission for Aerology established a Working Group on Synthesis of High-Atmosphere Data with the following terms of reference:

(a) To survey and collate the theoretical and observational evidence of the diurnal and semidiurnal waves in the fields of pressure, temperature, density and vector wind velocity as a function of latitude and season for altitudes up to 120 km;

(b) To collect and collate data on pressure, temperature, density and vector wind velocity for the layer 30-120 km with special attention to latitudinal, longitudinal and seasonal effects;

(c) To relate their findings to any recommendation of COSPAR on the international reference atmosphere;

(d) To consider the extent to which a tentative climatology of the high atmosphere can be established.

A session of the working group was held in Berkeley, California, U.S.A., in conjunction with the thirteenth session of the General Assembly of IUGG in August 1963.

Following its consideration of item (a) of its terms of reference, the group decided that a report should be prepared presenting an up-to-date brief statement of our present knowledge of tidal phenomena in the stratosphere and mesosphere, pointing out the regions where knowledge is lacking and recommending where future work is required. It was agreed that at present the observational and theoretical data are inadequate to produce a global method (e.g. tide-tables) of correcting upper-wind data and temperature observations for tidal effects and that only the most general statements can be made on this subject. It was also thought desirable to include a short account of the present knowledge on gravity waves in this region of the atmosphere. Professor B. Haurwitz, a member of the working group, who had prepared the first draft of this report, agreed to complete and revise it along the lines indicated above.

In view of the great interest and importance of this subject, the report of Professor Haurwitz in its revised form is now reproduced as a WMO Technical Note with the agreement of the president of the commission.

I am glad to have this opportunity of thanking Professor Haurwitz for having prepared this valuable report and the members of the group for the assistance they have given.

(D.A. Davies)
Secretary-General
TIDAL PHENOMENA IN THE UPPER ATMOSPHERE

Summary

Atmospheric tides, while small at the earth's surface compared to the weather variations, increase in intensity with elevation. Data obtained from radio tracking of meteor trails show that at heights between 80 and 100 km the solar-tidal wind oscillations are of the same magnitude as the mean winds, with pronounced seasonal variations. To interpret tidal data correctly it is important to have some statistical measure showing their reliability since they are affected by the presence of the irregular changes due to weather systems. Tidal theory attributes the magnitude of the solar semidiurnal oscillation ($S_2$) no longer to strong magnification by resonance, but rather to solar heating. It also indicates that the solar diurnal oscillation ($S_1$) will receive no magnification in the atmosphere, which will account for its comparative smallness, at least near the ground, compared with ($S_2$). The theory shows that the amplitude of the tidal oscillations like those of other gravity waves must increase by about two orders of magnitude between the surface and 100 km, but gives little other guidance concerning the distribution of the two largest oscillations $S_1$ and $S_2$ in the horizontal or vertical direction or with the season. Because of the practical importance of these two oscillations especially for the interpretation of high-level wind data an international programme to determine the tides in the upper atmosphere is required to obtain the missing information.
Les marées atmosphériques, qui sont faibles à la surface terrestre si on les compare aux variations météorologiques, s'intensifient avec l'altitude. Les données recueillies par le radiorepérage des traînées de météores montrent qu'entre 80 et 100 km les oscillations solaires du vent ont la même ampleur que les vents moyens et qu'elles sont l'objet de variations saisonnières très marquées. Pour l'interprétation correcte des données relatives aux marées atmosphériques, il importe de disposer de certains indices statistiques exprimant le degré de fidélité de ces données, puisqu'elles subissent l'influence de variations irrégulières dues aux systèmes météorologiques. La théorie des marées n'attribue plus l'ampleur de l'oscillation semi-diurne solaire ($S_2$) à une forte amplification par résonance, mais plutôt à l'action thermique du soleil. Elle indique également que l'oscillation diurne solaire ($S_1$) ne s'intensifie pas dans l'atmosphère, ce qui explique sa faiblesse relative par rapport à ($S_2$), du moins près de la surface terrestre. La théorie montre également que l'amplitude des oscillations des marées, de même que celle des autres ondes de gravité, doit environ doubler entre la surface et le niveau de 100 km, mais ne donne que peu d'autres indications sur la distribution des deux plus fortes oscillations $S_1$ et $S_2$ dans le sens horizontal ou vertical, ou en fonction de la saison. Étant donné l'importance pratique de ces deux oscillations, en particulier pour l'interprétation des données sur les vents à haute altitude, il est nécessaire de recueillir la documentation qui nous fait défaut grâce à un programme international d'études des marées dans la haute atmosphère.
ЯВЛЕНИЯ ПРИЛИВОВ В ВЕРХНЕЙ АТМОСФЕРЕ

КРАТКOE СОДЕРЖАНИЕ

Интенсивность атмосферных приливов, будучи незначительной по сравнению с вариациями погоды, с высотой возрастает. Данные, полученные с помощью радара при прослеживании следов метеоров, показывают, что на высоте от 80 до 100 км, солнечно-приливные ветровые колебания имеют тот же порядок, что и средние ветра с ярко-выраженными сезонными вариациями. Для правильного толкования данных о приливах важно иметь некоторое статистическое подтверждение того, что они являются надежными, так как на них оказывает влияние наличие нерегулярных изменений, вызываемых погодными системами. По теории приливов считается, что значительное возрастание солнечного полуусуточного колебания (S2) вызывается не резонансом, а скорее солнечным нагреванием. Эта теория показывает также, что солнечное суточное колебание (S1) не увеличивается в атмосфере, что объясняется его относительной незначительностью по сравнению с S2 по крайней мере у поверхности земли. Теория показывает, что амплитуда приливных колебаний, также как и в случае с другими гравитационными волнами, должна увеличиваться в слое между поверхностью земли и 100 км, примерно на два порядка. Однако эта теория мало что говорит о распределении двух самых больших колебаний S1 и S2 в горизонтальном или вертикальном направлении или по сезонам. В силу практического значения этих двух колебаний, особенно для интерпретации данных о ветре на больших высотах, необходимо разработать международную программу для определения приливов в верхней атмосфере, имеющую целью получение недостающей информации.
Resumen

Las mareas atmosféricas, que son muy débiles en la superficie terrestre comparadas con las variaciones de los elementos meteorológicos, aumentan en intensidad con la altitud. Los datos obtenidos al observar por radio las estelas de los meteoritos muestran que, a altitudes comprendidas entre los 80 y 100 Kms., las variaciones del viento debidas a las mareas solares son de la misma magnitud que el viento medio y varían considerablemente según la estación del año. Para interpretar correctamente los datos relativos a las mareas atmosféricas, es importante disponer de ciertos índices estadísticos que expresen el grado de fiabilidad de los mismos, puesto que estos datos experimentan la influencia de las variaciones irregulares de los elementos meteorológicos. En la actualidad, la teoría de las mareas no atribuye la amplitud de la oscilación semi-diurna solar ($S_2$) a una fuerte amplificación por resonancia, sino más bien a la acción térmica del sol. También indica la teoría que la oscilación diurna solar ($S_1$) no se intensifica en la atmósfera, lo que explica que su valor sea relativamente pequeño, por lo menos cerca de la superficie, en comparación con ($S_2$). La teoría muestra también que la amplitud de las oscilaciones de las mareas, como la de otras ondas de gravedad, debe aumentar hasta aproximadamente el doble de su valor entre la superficie y los 100 Kms. de altitud, pero no da indicación alguna sobre la distribución de las dos oscilaciones más fuertes $S_1$ y $S_2$ en el sentido horizontal o vertical, ni en función de la estación.

Debido a la importancia práctica de estas dos oscilaciones, especialmente para la interpretación de los datos de viento a gran altitud, es necesario obtener la información que falta por medio de un programa internacional para el estudio de las mareas en la atmósfera superior.
I. INTRODUCTION

The interest in atmospheric tides was at first largely theoretical because these periodic variations are very small near the earth's surface. But the advent of meteorological observations from higher levels, especially from the mesosphere and lower ionosphere, confirmed earlier theoretical predictions that these oscillations increase strongly upwards and attain amplitudes of the same magnitude as the non-periodic variations. This is true, in particular, for the high-level winds. It is therefore necessary to know the tidal variations in order to interpret correctly the individual observations made in the high atmosphere.

Another phenomenon, related theoretically to tides and affecting the state of the upper mesosphere and lower ionosphere is the presence of gravity waves. This subject is somewhat beyond the scope of the present paper, and only a brief discussion is given in Section VI for completeness.

Unfortunately, our knowledge of the air tides at high levels is at present neither theoretically nor observationally sufficient to forecast what the tides at a given moment and place should be and to construct tables as in the case of the ocean tides. The present survey can merely summarize what little is known up to approximately 100 km and give some very general information about tidal variations at high levels, in particular with regard to meteorological variables. It is hoped that this Note may give rise to the badly needed accumulation of more data by emphasizing the importance of the missing information.

For the following discussion it is desirable to recapitulate first the various periodicities which are called "atmospheric tides". This term refers generally to the regular variations of various meteorological parameters whose periods are fractions of a solar day (24 solar hours) or a lunar day (approximately 24.87 solar hours). For convenience these oscillations are denoted by $S_n$ for the oscillation whose period is the $n$th part of a solar day, and $I_n$ for the $n$th part of a lunar day. Of the lunar oscillations only $I_2$ has been found in the meteorological data. Among the solar oscillations $S_1$, $S_2$, $S_3$, and $S_4$ are of some interest. At the earth's surface only $S_2$ and perhaps $S_1$ can be directly recognized without statistical treatment of a large amount of data to eliminate the other superimposed, and generally much larger, variations referred to as "weather" or "meteorological noise". $S_2$ appears as a 12-hourly oscillation on tropical barograph traces with the pressure maxima occurring fairly uniformly everywhere at about 10 a.m. and 10 p.m. local time and with an amplitude of about 1 mb. The second largest oscillation is $S_1$ which can, in many cases, also be discerned in the daily barograph records. It has an amplitude about half of $S_2(p)$ and its global distribution is much less regular. These two pressure oscillations together represent the mean daily surface pressure variation quite well. As is well known, their combined effect has to be taken into account when pressure tendencies are to be interpreted for weather forecasts. The other oscillations, although of great theoretical interest, are so small, at least in the lower atmosphere, that they are of no practical significance.

Because the surface pressure oscillations, even $S_1$ and $S_2$, are so small the corresponding changes in such other meteorological parameters as wind, or density are equally insignificant at the earth's surface and can be disregarded for practical purposes.

It has been surmised for a long time that these oscillations of tidal character must be much larger at high altitudes. Theoretical considerations (see Section VI) suggest...
that the relative pressure amplitudes (the ratio of the oscillation amplitude to the ambient mean pressure), which even for $S_2$ is only about 0.1 per cent at the ground, and the other tidal variations, especially those of the wind should increase by about two orders of magnitude to an altitude of 100 km.

Indirect observational evidence for this increase of the tidal oscillations with height was first obtained through the daily variations of the geomagnetic parameters, both according to lunar and solar time. The dynamo theory, first proposed by Stewart in the Encyclopaedia Britannica in 1882, explains them as due to periodic motions of the ionized layers of the high atmosphere in the permanent magnetic field of the earth. (See, for instance, Chapman and Bartels, 1940, Chapter XXIII.) Unfortunately it has so far not been possible to obtain reliable quantitative values for the air motions in this manner because the parameters characterizing the electric state of the high atmospheric layers are not sufficiently well known. Likewise, the more recently observed variations of ionospheric parameters with tidal periods do not permit conclusive deductions of the corresponding variations of the meteorological variables such as pressure or wind, although they indicate strongly that the tidal variations in the high atmosphere must be much larger than at the ground.

The most satisfactory data on the atmospheric tides at high levels have come from radio meteor data. When a meteor penetrates the atmosphere to a level of about 80 to 100 km above the earth's surface it produces an ionized trail while it disintegrates because of its collision with the air molecules. This trail drifts along with the air motion. Radio signals are reflected from these drifting ionized trails, and the radial drift velocity with respect to the observer can be determined by Doppler techniques. Because of the great number of radio meteors it is possible to determine the wind velocity from these individual determinations of the radial velocities. The results of these observations are discussed in detail in Section IV. But to demonstrate the importance of the periodic variations Table I shows for Jodrell Bank (53.2°N, 2.3°W) (near Manchester), England and for Adelaide (34.9°S, 138.6°E), Australia, how often the mean wind $\bar{V}$ was larger or smaller than the amplitude of the 24-hourly, $A_1$, and 12-hourly $A_2$, component, both for the NS (meridional) and EW (zonal) components. The predominance of the periodic wind components is, of course, particularly striking for the meridional wind component, but even for the zonal component the importance of the periodic terms can be clearly seen. Obviously, wind observations in the high atmosphere at a given time and place must be greatly affected by the atmospheric tides.

Table I

<table>
<thead>
<tr>
<th></th>
<th>Jodrell Bank</th>
<th>Adelaide</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NS</td>
<td>EW</td>
</tr>
<tr>
<td>$\bar{V} &gt; A_1$</td>
<td>33</td>
<td>58</td>
</tr>
<tr>
<td>$\bar{V} &lt; A_1$</td>
<td>61</td>
<td>34</td>
</tr>
<tr>
<td>$\bar{V} &gt; A_2$</td>
<td>15</td>
<td>41</td>
</tr>
<tr>
<td>$\bar{V} &lt; A_2$</td>
<td>81</td>
<td>57</td>
</tr>
</tbody>
</table>
II. REPRESENTATION OF PERIODIC PHENOMENA

Even a meteorological parameter such as the temperature which has clearly a diurnal period shows, superimposed on this regular diurnal oscillation, other variations due to irregular weather effects. To eliminate these irregular variations the method of harmonic (or Fourier) analysis is used which need not be discussed here. (See Chapman and Bartels (1940), Chapter XVI; also Brooks and Carruthers (1953), for instance.) But since there is no uniformity in the presentation of the results of such harmonic analyses the relevant formulae are set down here to establish the framework in which the data on the high atmosphere will be presented. It is hoped that the following presentation may contribute toward an elimination of the existing confusion which leads to difficulties in interpretation, and even to mistakes.

Suppose that a variable \( p \), for instance the pressure, is a periodic function of time \( t \) (in angular measure) with a period whose length is \( 1/n \) of a day (\( n \) an integer). Then apart from the mean pressure which is of no interest here

\[
p = a_n \cos n t + b_n \sin n t
\]

\[
= A_n \sin (n t + \alpha_n)
\]

The two forms are entirely equivalent, and we have the following relations between the harmonic constants \( a_n, b_n \) and the amplitude \( A_n \) and phase angle \( \alpha_n \)

\[
A_n = (a_n^2 + b_n^2)^{1/2}
\]

\[
\alpha_n = \tan^{-1} \left( \frac{a_n}{b_n} \right)
\]

where the quadrant of \( \alpha_n \) can be determined without ambiguity because

\[
a_n = A_n \sin \alpha_n
\]

\[
b_n = A_n \cos \alpha_n
\]

In general, amplitude \( A_n \) and phase angle \( \alpha_n \) are the two constants of interest, but for many calculations with harmonic oscillations \( a_n \) and \( b_n \) are needed. While they can readily be computed if \( A_n \) and \( \alpha_n \) are known it is desirable that they be included in any listing of oscillations to facilitate further work with such data.

Forms using the cosine function, a negative phase angle, etc. are, of course, entirely equivalent to (1b) and can easily be transformed into it. It would, for instance, be convenient to use the form

\[
p_n = A_n \cos n (t - t_0)
\]

because this expression shows directly the time \( t_0 \) (in angular measure) at which \( p_n \) reaches

* Equations are numbered beginning with (1) in each section. Within each section equations are referred to merely by these numbers. An equation from another section is quoted by the number of that section and of the equation, for instance (V,2).
its maximum. The form (lb) is adopted here because it has been mostly used so far. It can easily be converted into (1c). If the quantity to be expressed is not a scalar, like the pressure, but a vector like the wind velocity its meridional and zonal components must be analysed separately. In general, the wind speed and direction are recorded so that first the two wind components have to be computed. In meteorology it is customary to call a wind by the direction from which it is blowing, but in ionospheric research the direction towards which the air moves is generally used to denote the wind. Clearly, this leads to all sorts of confusion. It is therefore strongly urged that it should always be stated which convention is used. The meteorological terminology practised by WMO is adopted throughout this paper. If ionospheric physicists and aeronomers feel that they cannot follow the WMO recommendation (which agrees with everyday usage) it would at least be helpful to indicate that the wind is named by the direction toward which it is blowing. For instance, a west wind could be called an "eastward" wind to avoid any ambiguity.

If the time $t$ is expressed in hours it must be multiplied by a factor 15 to convert into degrees, but this factor is often omitted. The time of the maximum, $t_{m,n}$, is related to the phase angle $\alpha_n$ by

$$t_{m,n} = \frac{90° - \alpha_n}{15n}$$

If $\alpha_n$ is larger than 90°, the 90° may be replaced by 45° to avoid negative values for $t_{m,n}$.

Since an oscillation is characterized by two numbers, $A_n$ and $\alpha_n$, or $a_n$ and $b_n$, it can be represented graphically by a point in a plane. Such a representation leads to a "harmonic dial", as illustrated in Figure 1 which shows the 12-hourly oscillation of the westerly component of the wind at about 80 - 100 km over Jodrell Bank, England (near Manchester) during the summer (comprising the months May, June, July, August). Here $b_2$ is the abscissa, $a_2$ the ordinate. The 35 individual determinations of this oscillation are represented by the data, the mean value by the cross connected with the origin. With our positioning of the Cartesian co-ordinates $a_2$, $b_2$ it is possible to indicate directly at which time the maximum occurs by considering the figure as a polar diagram. For the 12-hourly period the figure assumes then the same appearance as the face of a clock. Hence the name harmonic "dial". For a 24-hour period the hours indicated would, of course, run through 24 hours.

Figure 1 shows that the individual points scatter widely around the mean. This is not surprising since the individual determinations must be greatly affected by aperiodic day-to-day variation in the high atmosphere. It is therefore necessary to have a measure of the statistical reliability of the mean value derived from a number of individual determinations.

Two statistical quantities seem appropriate, the radius of the probable-error circle, $r$, and the expectancy $E$ whose ratio to the mean amplitude of the oscillation $A$ is a measure of the probability that an $A$ of the same magnitude may be found by harmonic analyses of random data. The subscript $n$ indicating the order of the harmonic is now omitted. A discussion of the underlying statistical theory can be found in Chapman-Bartels (1940), Chapter XVI, for instance, and only a brief description is given here (see also Bartels, 1932).
The data on which the harmonic analysis is to be performed are combined into \( m \) groups, the length of each group corresponding to a multiple of the period to be studied.

Each group may of course contain only one period. The harmonic coefficients of the oscillation as determined for each group are \( a_i, b_i \) \((i = 1, 2, \ldots, m)\). Then the amplitude for each group

\[
A_i^2 = a_i^2 + b_i^2
\]

The expectancy \( E \) is given by the expression

\[
E^2 = \frac{1}{m^2} \sum_{i=1}^{m} A_i^2
\]
Further, the mean value $\bar{A}$ of the amplitude for all data

$$\bar{A}^2 = \left(\frac{1}{m} \sum_{i=1}^{m} a_i \right)^2 + \left(\frac{1}{m} \sum_{i=1}^{m} b_i \right)^2 = \bar{a}^2 + \bar{b}^2$$  \hspace{1cm} (2)

Then the probability $q$ that for random data $\bar{A} \geq kE$ is

$$q = e^{-k^2}$$  \hspace{1cm} (3)

On the other hand, if

$$\delta a_i = a_i - \bar{a}, \quad \delta b_i = b_i - \bar{b}$$

the radius $r$ of the probable-error circle for $\bar{A}$

$$r = 0.83 \frac{1}{m} \left[ \sum_{i=1}^{m} (\delta a_i)^2 + \sum_{i=1}^{m} (\delta b_i)^2 \right]^{1/2}$$  \hspace{1cm} (4)

while the radius of the probable error circle for the individual determination, $r_i$, is $m^{1/2}$ times larger. Thus

$$k^2 = m(1 + 1.45 m r^2/\bar{a}^2)^{-1}$$  \hspace{1cm} (5)

It may be pointed out in passing that in the ideal case when the error vanishes $k^2 = m$. The mean amplitude $\bar{A}$ would then be $m^{1/2}$ times the expectancy $E$, and the probability that this occurs in random data is $e^{-m}$. This conclusion merely confirms what is known intuitively, namely that statistical tests and error computations based on few data are not satisfactory.

Chapman (1951) has suggested that a determination of harmonic coefficients must be considered unsatisfactory if $3\bar{r} \leq \bar{A}$. The probability that $3\bar{r} = \bar{A}$ for a number $m$ of groups is as follows:

<table>
<thead>
<tr>
<th>$m$</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>50</th>
<th>100</th>
<th>$\infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>1:15</td>
<td>1:46</td>
<td>1:74</td>
<td>1:114</td>
<td>1:147</td>
<td>1:252</td>
<td>1:384</td>
<td>1:500</td>
</tr>
</tbody>
</table>

For small $m$ when the statistical rules can hardly be applied the chance that Chapman's condition may be satisfied by random data is clearly not as small as one might wish. One would therefore, if at all possible, use a larger number of data. With $m = 10$ the possibility that an amplitude three times larger than the radius of the probable-error circle is found is only 1:46, with 20 only 1:114.

In Figure 1 the larger circle around the mean value is the probable error circle of the individual (group) determination, with its radius $r_i$. With a completely random distribution and a sufficiently large number half the data points should be inside that circle, half outside. In the example 16 points lie inside showing that the statistical conditions are reasonably well satisfied. The radius of the probable error circle of the mean value, $r$,
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is $3_{1/2}$ smaller, or $1.3$ m/sec while the amplitude is $7$ m/sec. Hence the mean value is well determined. From the computation of the expectancy it can be seen that the probability of finding a mean amplitude of such magnitude in random numbers is $e^{-12.7}$ or $3.1$ in $10^6$.

III. THEORY OF THE ATMOSPHERIC TIDES

Although the present review is mainly intended as a description and discussion of tidal phenomena in the high atmosphere a brief account of atmospheric-tidal theory is desirable for a better understanding of the tidal motions in the high atmosphere. Detailed accounts of the theory of the atmospheric tides have been given by Wilkes (1949), Chapman (1951), Kertz (1957), and Siebert (1961), and the reader is referred to their presentations for details.

The tidal force of the moon is 2.4 times that of the sun. Consequently, the ocean tides occur largely according to lunar time with maxima following each other every half lunar day (12 hours 26 minutes, approximately) rather than every half solar day. In the atmosphere $S_2$ is very much larger than $L_2$ as is demonstrated directly by the evidence of the daily surface pressure oscillation in the tropics with its period of 12 solar hours, rather than 12 lunar hours. To account for this fact Laplace suggested that $S_2$ is largely due to the sun's thermal rather than gravitational action on the earth's atmosphere. Solar heating and the diurnal temperature variation have not only a 24-hourly, but also a 12-hourly (and 8- and 6-hourly) period because of the asymmetry of the daily temperature curve. But the diurnal term in the daily temperature wave is about 2.5 times that of the semidiurnal term (Haurwitz, 1962a). In order to account for the fact that, nevertheless, the semidiurnal oscillation is larger than the diurnal oscillation Kelvin suggested that the atmosphere has a free period in the vicinity of 12 hours so that the period, but not the 24-hourly or the lunar semidiurnal period, is greatly magnified. This explanation has since become known as the "resonance" theory. It requires a temperature maximum of approximately 350 K near the stratopause as has been shown by Pekeris (1937). We know now that the temperature at this level is well below these requirements of the resonance theory. With realistic temperature distributions the magnification becomes only about three-fold. There is no pronounced resonance peak at 12 hours. The lunar-tidal magnification is similarly small, in agreement with the observed value for the lunar tide, and the solar diurnal tide ($S_1$) receives practically no magnification.

To explain why $S_2$ is larger than $L_2$ and $S_1$ Siebert (1961) and Sen and White (1955) pointed out that the earlier explanations of the semidiurnal oscillation on a thermal or partly thermal basis considered only that part of the diurnal heat wave which is propagated by eddy conductivity from the ground upward to about 500 m. But even at higher levels considerable daily variations of the air temperature must occur because of the absorption of radiation. Thus a much larger mass of the atmosphere is affected by this heating and a much smaller magnification is required than assumed by the original resonance theory. It can easily be surmised that the assumption of a temperature variation extending through the whole atmosphere reduces drastically the requirement for resonance magnification. The lowest 500 m comprise only 1/16 of the mass of the whole atmosphere, so that a uniform temperature variation extending through the whole atmosphere would make the required magnification much smaller. According to the resonance theory the magnification of the semidiurnal pressure oscillation should be 80 to 100 times. The reduction would bring it down to 5 to 6 times, comparable with the modest magnification given by a realistic atmospheric model.

It remains to explain why the diurnal oscillation is smaller than the semidiurnal oscillation. To consider this problem the global distributions of $S_1$ and $S_2$ both for
pressure and temperature have to be described first in somewhat more detail. A study of the
global surface temperature variation has shown that on a world-wide basis the semi-diurnal
temperature variation has an amplitude about $\frac{4}{10}$ of the diurnal variation (Haurwitz, 1962a).
The semi-diurnal oscillation $S_2$ of the surface pressure $p_0$ in mb can be well approximated by
the empirical formula (Haurwitz, 1956; see also Simpson, 1918)

$$S_2(p_0) = 1.16 \sin^3 \theta \sin(2t + 158^\circ) + 0.06 \cos (\pi \cos \theta) \sin(2t - 21^\circ + 303^\circ).$$

Here $\theta$ denotes the colatitude, $t$ the local mean time in angular measure, $\lambda$ the longitude.

The solar semi-diurnal oscillation is thus seen to consist of two parts. The larger
component is a wave travelling westward whose maximum occurs everywhere shortly before 10
hours local mean time as shown by the phase angle of $158^\circ$. Its amplitude decreases from the
equator towards both poles; but the dependence on the colatitude as given here is only an
empirical form, convenient for many estimates while the theory gives a more complicated func-
tion consisting of a series of associated Legendre functions.

The smaller component is a standing oscillation since its phase does not depend on
local mean time $t$, but on Greenwich mean time, $t - \lambda$. It is significant only at high
latitudes where the travelling wave is small. The solar tidal potential contains no term
which can produce a standing oscillation. Its cause must therefore be the sun's heating
effect. Moreover, as the dependence on Greenwich time shows, this cause must be fixed
relative to the earth, and it is in fact due to the unequal heating of the irregularly
distributed water and land masses.

The geographical distribution of the diurnal (24-hourly) surface pressure oscillation $S_1(p_0)$ is much less regular than that of the semi-diurnal oscillation. Orographic
effects, and especially the land and water distribution, clearly disturb $S_2(p_0)$ much more
than $S_2(p_0)$. Nevertheless the diurnal temperature wave must produce a world-wide diurnal
surface pressure oscillation travelling westward which may be called the "planetary" compo-
nent of the diurnal oscillation on which the more local disturbances are superimposed. To
determine this planetary component of $S_1(p_0)$ mb an analysis of the available data has been
performed resulting in the following approximate formula

$$S_1(p_0) = 0.59 \sin^3 \theta \sin(t + 12^\circ).$$

The amplitude of $S_1(p_0)$ is thus about half that of $S_2(p_0)$, and the diurnal maximum occurs at
5.2 hours a.m., local mean time.

As mentioned above, tidal theory (Eiebert, 1961) indicates that the diurnal oscil-
lation should not be magnified at all in the earth's atmosphere. Since the amplitude of the
semi-diurnal temperature variation is $\frac{4}{10}$ of that of the diurnal variation, and since the
semi-diurnal oscillation in our atmosphere is magnified about threefold, the semi-diurnal pres-
sure oscillation should have an amplitude at least 1.2 times that of the diurnal oscillation
while actually it is twofold. In view of the approximations involved in these crude esti-
mates this difference does not seem to be crucial. Thus the main problem requiring explana-
tion appears to be the very regular global distribution of $S_2$ as contrasted to that of $S_1$.

It has been suggested (Small and Butler, 1963) that the main cause of $S_2$ is the
daily temperature change in the upper ozonosphere. Now, both the case of strong resonance
and the case when the main cause of the oscillation is situated aloft require that the
Table II
Wind Oscillations at Jodrell Bank and Adelaide at 80 - 100 km

<table>
<thead>
<tr>
<th>Season</th>
<th>N</th>
<th>u (northerly component)</th>
<th>m/s</th>
<th>[276x464]v (westerly component)</th>
<th>m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a</td>
<td>b</td>
<td>A m/s</td>
<td>t, m/s</td>
<td>α (±)</td>
</tr>
<tr>
<td>Diurnal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jodrell Bank</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>33</td>
<td>6.2</td>
<td>0.9</td>
<td>6.3</td>
<td>+</td>
</tr>
<tr>
<td>D</td>
<td>26</td>
<td>-2.9</td>
<td>-1.7</td>
<td>3.3</td>
<td>1.7</td>
</tr>
<tr>
<td>J</td>
<td>35</td>
<td>8.5</td>
<td>-0.3</td>
<td>8.5</td>
<td>1.6</td>
</tr>
<tr>
<td>Total</td>
<td>94</td>
<td>4.6</td>
<td>-0.2</td>
<td>4.6</td>
<td>--</td>
</tr>
<tr>
<td>Adelaide</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>14</td>
<td>6.4</td>
<td>17.5</td>
<td>18.6</td>
<td>4.7</td>
</tr>
<tr>
<td>D</td>
<td>12</td>
<td>-10.9</td>
<td>16.9</td>
<td>20.2</td>
<td>4.4</td>
</tr>
<tr>
<td>J</td>
<td>11</td>
<td>7.1</td>
<td>10.1</td>
<td>12.3</td>
<td>4.0</td>
</tr>
<tr>
<td>Total</td>
<td>37</td>
<td>0.8</td>
<td>15.1</td>
<td>15.1</td>
<td>--</td>
</tr>
<tr>
<td>Semidiurnal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jodrell Bank</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>35</td>
<td>1.6</td>
<td>-7.1</td>
<td>7.2</td>
<td>2.2</td>
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<tr>
<td>D</td>
<td>28</td>
<td>18.7</td>
<td>5.4</td>
<td>19.5</td>
<td>2.3</td>
</tr>
<tr>
<td>J</td>
<td>35</td>
<td>8.4</td>
<td>-8.1</td>
<td>11.6</td>
<td>1.3</td>
</tr>
<tr>
<td>Total</td>
<td>98</td>
<td>8.7</td>
<td>-3.9</td>
<td>9.5</td>
<td>--</td>
</tr>
<tr>
<td>Adelaide</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>14</td>
<td>-4.5</td>
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<tr>
<td>D</td>
<td>12</td>
<td>-15.2</td>
<td>-5.2</td>
<td>16.1</td>
<td>3.7</td>
</tr>
<tr>
<td>J</td>
<td>11</td>
<td>4.6</td>
<td>-10.0</td>
<td>11.0</td>
<td>4.0</td>
</tr>
<tr>
<td>Total</td>
<td>37</td>
<td>-5.2</td>
<td>-5.8</td>
<td>7.8</td>
<td>--</td>
</tr>
</tbody>
</table>
amplitude of the pressure and wind oscillations should become zero at some intermediate level. At this height a phase shift of 180° should also occur. Unfortunately, no observations are available to clear up this theoretically important point. It will be considered in Section V with the few available data from the upper stratosphere and mesosphere.

**IV. WINDS IN THE HIGH ATMOSPHERE**

Because of the expensive techniques required, data on tidal oscillations in the high atmosphere are still very sparse and do not give a clear and comprehensive picture of the distribution of these oscillations. The discussion here is based to a large extent on results published for Jodrell Bank, England by Greenhow and Neufeld (1961) and for Adelaide, Australia by Elford (1959). These observations were made by radio measurements of the drift of ionized meteor trails as explained in Section I.

Harmonic coefficients for diurnal and semidiurnal winds have also been published for a few other stations. These have been summarized elsewhere (Haurwitz, 1962b). Most of these results are obtained from drift observations of E layer inhomogeneities, and it is therefore even doubtful if they represent true air motions. All that can be said is that these observations confirm the order of magnitude of the periodic wind variations at these levels, and the existence of much stronger seasonal variations than at low altitudes.

Since the tidal variations show a seasonal variation it is customary to combine them in three groups of four months each, unless sufficient data are available to allow a statistical treatment of monthly means. These three groups with their identifying letters are

- **Equinoctial months**: March, April, September, October
- **D months**: November, December, January, February
- **J months**: May, June, July August

These groups are also used here.

The results of these wind determinations are shown in Table II. Here \( n \) denotes the number of individual determinations, \( a \) and \( b \) the cosine and sine factors, \( A \) the amplitude in m/sec, \( r \) the radius of the probable error circle of the mean in m/sec, \( \alpha \) the phase angle, and \( t \) the time when the maximum occurs. The error limits of the angle can be obtained from \( r \) and \( A \) and are given by

\[
\pm \sin^{-1} \left( \frac{r}{A} \right)
\]

In those cases when \( r > A \) the phase angle is completely undetermined.

The data contained in Table II are shown graphically in Figures 2 and 3 in harmonic dials. The annual mean values are here represented by crosses. Probable error circles have not been determined for these annual means because they can be estimated from the seasonal values. Greenhow and Neufeld (1961) have discussed their Jodrell Bank observations in considerable detail. Besides \( S_1 \) and \( S_2 \), they find also a small 8-hourly variation, \( S_3 \), whose mathematical expression is for the northerly component in m/sec

\[
u = 2.5 \sin (3t + 292.5°)
\]

for the westerly component in m/sec

\[
v = 3.1 \text{ m/sec} \sin (3t + 45°)
\]

This wind oscillation at high levels does not seem to show the seasonal phase reversal.
Figure 2

Harmonic dials for the mean northerly and westerly components of the diurnal (a) and semidiurnal (b) wind components at Jodrell Bank at 80-100 km. Crosses indicate annual mean values, dots seasonal values. Circles show probable errors of the seasonal means.
Figure 3
Harmonic dials for the mean northerly and westerly components of the diurnal (a) and semidiurnal (b) wind components at Adelaide, Australia at 80-100 km. Crosses indicate annual mean values, dots seasonal values. Circles show probable errors of the seasonal means.
characteristic for the S-hourly oscillation at the surface in extratropical latitudes. Probable errors are not given for $S_3$.

To illustrate further the behaviour of the periodic part of the wind, hodographs are shown in Figure 4 for the three seasons at Jodrell Bank and at Adelaide. Only the periodic component consisting of the diurnal and semidiurnal part is shown, but not the mean wind. The periodic wind blows in the direction from the centre to the hodograph curve on which every second hour (in local mean time) is indicated by a dot. Depending on the relative magnitudes of the diurnal and semidiurnal oscillations the hodographs have very different shapes. In general, the wind vectors turn in the theoretically expected direction, i.e. with the sun. Only during the J months in Adelaide during part of the day is the rotation in the wrong direction. This deviation from the theory may be due to the very poor determination of the diurnal westerly wind component whose probable-error circle has a radius larger than the amplitude (Figure 3a and Table II).

At Jodrell Bank $S_2$ has clearly a much larger amplitude during the winter than during the summer, and the same is true for Adelaide if only the observations in the layer from 85 to 94 km are considered. But if the observations from somewhat lower (75-84 km) and higher (95-104 km) layers are added as has been done here the seasonal differences become much smaller, especially if allowance is made for the probable errors.

Other data based on observations of the drift of ionized meteor trails have been obtained at Kharkov, U.S.S.R. by Kashcheyev and Lissenko (1961) for the months March through June. A comparison with the Jodrell Bank data is difficult because no measure of the variability of the Kharkov data is available, but, as at Jodrell Bank, $S_2$ is larger than $S_1$.

Both oscillations, $S_1$ and $S_2$, may change rapidly with elevation between 80 and 100 km as is also suggested by some theoretical calculations. According to Greenhow and Neufeld (1961) the mean gradient of $S_2$ between 85 and 100 km is $2 \text{ m sec}^{-1} \text{ km}^{-1}$ during winter, and zero during summer so that in the yearly average the amplitude would be about 15 m/sec larger at 100 km than at 85 km. The phase difference, according to the same authors, is such that the oscillation at 100 km leads that at 85 km by about 3.5 hours during the winter, and by 1.5 hours in summer. But it is not to be expected that the values, even if they hold for Jodrell Bank can be applied in more than the most qualitative fashion to other localities.

For the extended layer between the top of the mesosphere and 25 to 30 km height only some very sporadic data based on very few observations are available. Lenhard (1963) computed the diurnal and semidiurnal wind oscillation over Eglin Air Force Base (30.5°N, 86.5°W) at 35, 43 and 64 km. Only observations within one single 24-hour period were available so that these determinations cannot be considered representative. Nevertheless, it may be mentioned that according to Lenhard's computation the amplitude of the wind oscillations decreases downward as would be expected on theoretical grounds, and that at 35 km it is of the order of 4 m/sec for the diurnal, and 2 m/sec for the semidiurnal oscillation (Lenhard, 1963).

From the surface to the 15-mb level values of the diurnal and semidiurnal oscillations of the wind have been obtained by Harris et al. (1962) for Lajes Field, Terceira, Azores (39.7°N, 27.1°W). Through this altitude range the amplitudes of these oscillations are less than 1 m/sec and therefore of no practical importance. For England, Johnson (1953) has determined the diurnal and semidiurnal oscillation of the wind at the 100 mb surface and found values similar to those of Harris et al. for the amplitudes and, in the case of $S_2$, for the phase angles. The determinations for these two stations are compared in Table III.
Figure 4 (a-d) - Hodographs of the periodic (diurnal plus semidiurnal) part of the daily wind variation for different seasons at Jodrell Bank (a-c) and Adelaide (d). The wind blows from the origin to the points on the hodograph curve. Hours indicated by dots on the curves.
Figure 4(e) - Hodographs of the periodic (diurnal plus semidiurnal) part of the daily wind variation for different seasons at Adelaide. The wind blows from the origin to the points on the hodograph curve. Hours indicated by dots on the curve.

Table III

Wind oscillations at 100 mb after Harris et al. and Johnson

<table>
<thead>
<tr>
<th>n</th>
<th>Northerly component</th>
<th>Westerly component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A cm/sec</td>
<td>α</td>
</tr>
<tr>
<td>Harris 1</td>
<td>34 ± 11</td>
<td>0</td>
</tr>
<tr>
<td>Johnson</td>
<td>28</td>
<td>122</td>
</tr>
<tr>
<td>Harris 2</td>
<td>28 ± 6</td>
<td>253</td>
</tr>
<tr>
<td>Johnson</td>
<td>34</td>
<td>224</td>
</tr>
</tbody>
</table>

A lunar semidiurnal oscillation of the winds in the lower E region with amplitudes of 15 to 20 m/sec has been reported on the basis of ionospheric drift data by J.H. Chapman (1953) and Phillips (Briggs and Spencer, 1954). But because of the method of observation it is doubtful that these variations really represent air motions. Greenhow and Neufeld (1961) found from an analysis of their data that the lunar tidal amplitude cannot be larger than
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2 m/sec. Matsushita (1964) concluded from a critical analysis of the lunar-day geomagnetic observation that

\[ u = 1.7 \sin \theta \cos \theta \sin (2\tau + 60^\circ) \]

\[ v = 1.7 \sin \theta \sin (2\tau + 150^\circ) \]

where \( \theta \) is the colatitude, \( \tau \) mean lunar time and the wind components are in m/sec. It is difficult to assign an exact altitude to these winds, but they are presumably required to blow in the lower E region around 100 km. It thus appears on balance that for practical purposes the lunar tidal wind variation can be disregarded in the layers with which we are concerned here.

V. PRESSURE AND DENSITY OSCILLATIONS

As mentioned before, ionospheric and geomagnetic parameters show oscillations of tidal character, but their interpretation in terms of meteorological parameters is difficult. As an example, an oscillation with the lunar semidiurnal period has been found by various authors (for instance Thomas and Svensson, 1955) for the equivalent height of the E (or E\(_s\)) layer. The amplitude of this oscillation seems to lie between 0.5 and 1 km. If this height variation \( \delta h \) may be regarded as that of an isopycnic or isobaric surface one has, because of the small percentual temperature variation, approximately

\[ \frac{\delta h}{H} = \frac{\delta p}{p} = \frac{\delta p}{\rho} \]

where \( H \) is the scale height, \( \rho \) the density. If \( H = 7 \) at 100 km one would expect a pressure amplitude of about 7 to 15 per cent at this level as compared to less than 0.01 per cent at the ground. The values for the lunar tidal wind velocity estimated by Matsushita (1964) would indicate a ten times smaller percentual pressure amplitude. On the other hand, the discrepancy between the two values could be reconciled by assuming that they refer to different altitudes, since theoretical calculations make it plausible that the lunar tidal amplitude increases rapidly in these layers (Sawada, 1954).

For \( S_1 \) and \( S_2 \) at 96 km Greenhow and Hall (1960) have found an amplitude of the percentual density variation of 13 per cent for the diurnal oscillation, and about 5 per cent for the semidiurnal oscillation. The determinations were made during winter at Jodrell Bank from observations of the diffusion of radio meteor trains. From the diffusion coefficients the air density can be deduced.

Another way of deriving values of the pressure oscillations at these heights is from the wind oscillations. Let \( u_n \) and \( v_n \) denote the oscillations with period \( \frac{1}{n} \) of a day of the northerly (southward) and westerly (eastward) component of the wind, \( p_n \) the pressure oscillation, \( \rho \) the mean density, \( \lambda \) the geographic longitude, \( \theta \) the colatitude, \( a \) the earth's radius, \( \omega \) the angular velocity of the earth's rotation. Then

\[ \frac{\delta u_n}{\delta t} - 2 \omega \cos \theta v_n = -\rho^{-1} \frac{\partial p_n}{\partial \theta} \quad (1) \]

\[ \frac{\delta v_n}{\delta t} + 2 \omega \cos \theta u_n = -\rho^{-1} \frac{\partial p_n}{\partial \sin \theta} \lambda \quad (2) \]

These equations hold under certain simplifying assumptions (e.g. that friction and the nonlinear terms can be neglected), which are probably not too badly satisfied up to these
levels. Since both the dependence of pressure and wind components on $t$ and on $\lambda$ are known one can use equation (2) to determine $p_n$ from $u_n$ and $v_n$. If an assumption is also made about the dependence of $u_n$, $v_n$, and $p_n$ on $\theta$ one can obtain two equations from which to derive $p_n$, and thereby a possibility of estimating better the reliability of the computed pressure oscillation.

It is assumed here that the amplitude of $S_2(p)$ is proportional to $\sin^3 \theta$ which agrees well with the surface data (Simpson, 1918; Haurwitz, 1956) and that the amplitude of $S_1(p)$ is proportional to $\sin \theta$ which presumably approximates at least roughly the latitudinal distribution of the solar heating which causes the diurnal oscillation, although at the surface $S_1(p)$ appears to be better approximated by $\sin^3 \theta$. Then one obtains from the wind observations at Jodrell Bank and Adelaide the pressure oscillations shown in Figures 5 and 6. The amplitudes are here shown reduced to their equatorial values by multiplication with the appropriate powers of $\sin \theta$ to make the two stations comparable and are expressed in percentages of the mean pressure at the level of observation. The values are given for each season, and the annual means computed from the annual mean oscillations of the two wind components are indicated by crosses. Each point represents the mean of the values obtained from the two wind components. Figure 5 shows that for the 24-hourly oscillation the amplitude is larger during the summer than during the winter, as might be expected, but the equinoctial value is about as large as the summer value. The phase angles for Jodrell Bank and Adelaide differ greatly, even though one would expect at both stations the same dependence on the sun's zenith angle since the pressure oscillation is caused by the sun. Also the large phase shift from winter to the equinoxes and summer at Jodrell Bank is unexpected and very likely not real. For the semidiurnal pressure oscillation (Figure 6) the seasonal variations are also large. The equinoxes have the smallest values, but the largest amplitudes in both hemispheres seem to occur during the D months, although the change from the D to J months is small in the case of Adelaide and within the error limits.

Two important features are brought out by these two figures. First there is the large percentage variation of the pressure throughout the day, as much as 8 per cent for the semidiurnal oscillation at Jodrell Bank during the D months and nearly 4 per cent for the diurnal oscillation at Adelaide during the E months. Even if it is remembered that these are values reduced to the equator by $\sin \theta$ and $\sin^3 \theta$, respectively, for $S_1$ and $S_2$, these percentage variations if applied to a surface pressure of 1000 mb would be of the same magnitude as the synoptic pressure changes and much more rapid. Secondly, while it has been suggested that at these high levels the phase of the semidiurnal pressure oscillation should be opposite to its surface value, 158°, Figure 6 shows that the observations to date do not bear out this suggestion.

As in the case of the wind oscillations information about the pressure oscillations for the layers between about 30 and 80 km is practically non-existent. For the layer between 30 km and the ground data have been published for $S_1$ and $S_2$ by Harris et al. (1962) for Lajes Field in the Azores. Figure 7 shows a smoothed version of their results for the relative pressure amplitude of $S_1$ with the mean pressure as height co-ordinate. The small circle indicates the value computed from Johnson's (1953) wind data for England. At the 0.01 mb level the annual mean value computed for Jodrell Bank (J.B.), actually valid for a level about 15 km higher, has been indicated. The relative pressure amplitudes computed for Jodrell Bank from the two wind components differ more than those for the lower atmosphere over England, and the spread is indicated by the thick horizontal line. The relative amplitude of the diurnal pressure oscillation appears thus to increase about a hundredfold from the ground to the 10 mb level, but its further increase upwards is much smaller.
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Figure 5 -
Harmonic dial showing the relative pressure amplitudes (amplitude as percentage of mean pressure) at Jodrell Bank and Adelaide between 80 and 100 km for the diurnal pressure oscillation. Crosses indicate annual mean values.

Figure 6 -
Harmonic dial showing the relative pressure amplitudes (amplitude as percentage of mean pressure) at Jodrell Bank and Adelaide between 80 and 100 km for the semidiurnal pressure oscillation. Crosses indicate annual mean values.
It must be mentioned here that the pressure data for $S_1(p)$ up to 30 km, as well as those for $S_2(p)$ to be discussed below, may well be subject to errors, especially in the stratosphere where radiational errors of the temperature sensors may make themselves felt. (I am greatly indebted to Messrs. Teweles and Harris for extensive discussions on these points.) If $S_1(p)$ is computed from the wind data for the Azores pressure amplitudes for $3_{1} (p)$ about one-third smaller than those shown in Figure 7 are obtained above the 200 mb level. But it is by no means clear that the values computed from the winds are more accurate than those given in Figure 7. The amplitudes derived from the wind data show very irregular changes with altitude indicating that considerable inaccuracies may be involved in their computation. The point indicating the amplitudes determined from the winds over England is between those determined for Lajes Field directly and from the winds. In any case, it can be assumed that Figure 7 gives at least the right order of magnitude, although the actual values may be too large by a factor 3, and the increase with altitude up to 30 km may be less than suggested at the end of the last paragraph.

The data of Harris et al. for $S_2(p)$ are shown in Figure 8, again in smoothed form. The short horizontal lines indicate the magnitude of the diameter of the probable error circle. Three values from other stations have been added. One is a determination for 200 mb based on the station pair Guam (13.4°N, 144.6°E) - Kwajalein (9.6°N, 167.6°E); the others, for 175 mb and 250 mb, are based on the station pair Coco (28.1°N, 80.4°W) - Long Beach.
Relative amplitude (ratio of amplitude to mean pressure) of the semi-diurnal pressure oscillation over the Azores after Harris et al. (1962). The short horizontal lines show the magnitude of the probable error.

G refers to data from Guam
E = Eglin Air Force Base, Florida
C to data from Cooc
J = Johnston Island
J.B. = Jodrell Bank.
Since only four daily observations were made at these stations, less than required to determine the second harmonic, and since $S_2$ migrates with a presumably constant speed westward one may treat two stations at similar latitudes as one station and their longitude difference as a time difference. But this procedure is clearly only a stop-gap method. The values for these stations have been reduced to the latitude of the Azores by the factor $\sin^3 \theta$ to make them comparable. While Guam fits very well with the Azores value, Coco is a bit larger, which just demonstrates again that not too much reliance must be placed on the results of one station.

The value of the amplitude of $S_2(p)$ at the Azores at 10 mb has a very large probable error so that it is impossible to be certain that the decrease of the amplitude at this level is real. Such a decrease would be of great interest since one would expect a nodal point at about this altitude (Haurwitz, 1962b) if the main cause of the semidiurnal oscillation is at higher levels rather than near the surface (for instance, daily temperature oscillation in the ozone layer). (See Section III.)

To fill the observational void in the upper stratosphere and mesosphere the semidiurnal pressure amplitudes have been computed from the wind observations at Eglin Air Force Base (30.5°N, 86.5°W) at 64, 43, 25 km, marked E (Lenhard, 1963). An additional point is obtained from a paper by Smith (1960) who determined the diurnal and semidiurnal wind oscillations over Johnston Island (18°N, 169°W) at 73, 76, and 79 km from 22 observations taken over a period of three weeks. Because of the day-to-day variations of the atmosphere no reliability can be expected from such a determination, and it is not surprising that the periodic wind vectors over Johnston Island turn in the right sense only at the 79 km level. Therefore, only the results for the 79 km level have been used here to compute the corresponding pressure amplitude, marked J in Figure 8. The value computed from the Jodrell Bank wind data (Greenhow and Neufeld, 1961) is indicated by J.B. The two wind components give somewhat different pressure amplitudes, and the difference is shown by the short horizontal lines.

As has been pointed out before, in the case of $S_1(p)$, some question arises concerning the accuracy of the Azores data because of possible radiational errors of the temperature sensors. If the amplitudes of $S_2(p)$ are computed from the wind component values different from those of Figure 8 are obtained. However, the differences are not systematically too large. Thus the points shown in Figure 8 are probably quite reasonable for conditions over the Azores. But it must be emphasized again that it is not clear how representative conditions over the Azores are for other locations, and that the values from Eglin Air Force Base and Johnston Island are based on so few data that they can at the very best give only an indication of the order of magnitude. Even so the values for Johnston Island appear too large. To allow for the latitude variation of the amplitude of the semidiurnal pressure variation all values have been reduced to the latitude of the Azores by using the factor $\sin^3 \theta$.

VI. INTERNAL GRAVITY WAVES

Apart from the tidal oscillations and meteorological disturbances with larger time scales the high atmosphere at 80 km and above shows also wind fluctuations of considerably shorter duration and smaller spatial scale. As in the case of the tidal oscillations these fluctuations are mainly determined from meteor observations, both by radio and visual methods. They seem to be confirmed by the observations of sodium vapour clouds released artificially into the atmosphere. An explanation of these short-period and small-scale variations was first sought on the basis of turbulence theory. But more recently, through the work of Hines (1960, 1963) it has become highly probable that these phenomena must be attributed to internal gravity waves in the atmosphere.
These internal gravity waves are of much lower frequency than the sound waves. The wave motion is essentially transverse to the direction of phase propagation rather than longitudinal as for sound waves. The shortest possible frequency of these waves is given by the Brunt-Väisälä period,

$$\tau_v = 2\pi T g^{-\frac{1}{2}} \left( \frac{\varepsilon_{ad}}{\varepsilon} \right)^{-\frac{1}{2}}.$$  

Here $T$ is the temperature, $g$ the acceleration of gravity, $\varepsilon_{ad}$ the adiabatic rate of temperature change, $\varepsilon$ the local rate of temperature decrease with elevation. Physically this expression represents the period with which a displaced air parcel would oscillate in the vertical when frictional effects are neglected and no account is taken of the condition of mass continuity. With $\varepsilon$ equal to zero the Brunt-Väisälä period would be about 4.5 minutes at 90 km altitude. The observed dominant periods of the short-time fluctuations at these altitudes are of the order of 100 minutes or more and thus well above the shortest permissible period.

It should be pointed out here that these internal gravity waves are different from the internal waves at the interface of two fluids of unequal density, although the two are closely related since gravity plays an important role in the origin and maintenance of both wave types.

From theory it follows that for the wave modes of interest at around 90 km, the ratio of the horizontal to the vertical wavelength is similar to the ratio of the actual period $\tau$ to the limiting Brunt-Väisälä period $\tau_v$. Since this ratio is of the order of 20 or more according to the above figures, the hypothesis of internal gravity waves can account for the observation that the horizontal scale size of these fluctuations exceeds the vertical size by a factor 20 or more. Observations suggest for the dominant mode at 90 km a vertical wavelength of about 12 km.

Theoretically the ratio of the horizontal to the vertical velocity due to the wave motion is nearly equal to the ratio of the horizontal to the vertical wavelength. This is in agreement with the empirical fact that the dominant observed motions are nearly horizontal.

The amplitudes of the internal gravity waves increase with elevation, just like the tidal waves which are, in fact, a special type of gravity waves of low-frequency. The reason for this increase can be understood if the upward flux of energy is considered. Since this flux must remain nearly constant, except where reflection of the waves takes place, the velocity must increase with the elevation to compensate for the decrease of the density with altitude. The observational evidence indicates that the intensity of the irregular winds does indeed increase with the altitude.

The wave motions are subject to the dissipative effects of molecular viscosity and thermal diffusivity both of which become important under similar conditions. Hines considers in particular the effect of viscosity which depends on the magnitude of the coefficient of kinematic viscosity. Since this coefficient is very nearly inversely proportional to the ambient density, the influence of viscous dissipation increases with height. On the other hand, viscous dissipation is more significant, the smaller the scale size. Thus, viscosity can be expected to obliterate the smaller scale sizes as the spectrum of internal waves progresses upwards through the atmosphere, a conclusion that appears to be supported by the available observations.

Because of the irregularity of the observed wind fluctuations a broad spectrum of internal gravity waves must exist. A theoretical explanation of the spectrum depends on a knowledge of the mechanism generating the waves. Various possibilities have been suggested. One of these considers non-linear effects on the tidal waves at ionospheric altitudes where the non-linear terms in the hydrodynamic equations can no longer be neglected. The non-linearity of the process may give rise to higher harmonics so that, similar to the concept of turbulence theory, energy cascades downward into the smaller scales.
Another possibility is that the energy of these waves is propagated upwards from weather systems or orographically induced disturbances existing in the troposphere and lower stratosphere. But it is not yet clear whether such an energy transport is possible with the existing mean wind and temperature distributions. Finally, it seems that even in the mesosphere and at meteor heights meteorological disturbances of similar magnitude and intensity as in the lower atmosphere appear whose energy may be responsible for the internal gravity waves. It is, of course, entirely possible that all three mechanisms contribute to the energy of the waves.

Although the waves, especially those of smaller scales, suffer dissipation during their propagation upward, the larger systems may very well penetrate well up into the higher ionosphere and give rise to the quasi-harmonic oscillations called "travelling ionospheric disturbances" in the F-layer.

VII. CONCLUSIONS

As predicted by the theory the atmospheric tidal oscillations increase by about two orders of magnitude from the ground to 80 - 100 km elevation. Thus the solar tidal wind oscillations at those levels are of the same intensity as the mean winds and play an important part in the circulation of the high atmosphere. The relative pressure amplitude, that is the ratio of the amplitude of the oscillation to the mean ambient pressure, shows a similar increase with altitude. While it is about 0.1 per cent or less at the ground it reaches values up to 10 per cent at 100 km so that the daily barometric variations in these layers are relatively larger than those experienced near the ground with the passage of deep cyclones.

The seasonal variations of the solar tidal oscillations at meteor heights are much more pronounced than at the ground. From the limited data it appears that $S_2$ is strongest during the winter months and weakest during the equinoxes. $S_1$, on the other hand, appears to be strongest in summer, weakest in winter, with the equinoxes intermediate. Both oscillations undergo also considerable phase shifts from season to season.

While at the ground $S_2$ is better developed than $S_1$, at meteor heights both are about equally important.

The lunar semidiurnal oscillation $L_2$ is at meteor heights, just as at the earth's surface, considerably smaller than $S_2$, and it has not yet been reliably determined from meteorological variables, although it has been established in ionospheric characteristics.

The main conclusion of this report is unfortunately that the present state of our knowledge of tidal phenomena in the high atmosphere is still very unsatisfactory, both with regard to the further development of the theory and for practical uses in interpreting meteorological observations at great altitudes. In order to remedy this situation much more systematic observational work is needed. Specifically, the following recommendations are made to improve our knowledge of the atmospheric tides at greater heights.

(1) More uniformity in the presentation of data will greatly facilitate the use of these data. The following conventions are recommended:

(1) Winds should be named by the direction from which they are blowing (see Resolution 15 (EC-XIV)). To avoid confusion the nomenclature should be explained, even when this convention is adopted.

(11) To facilitate comparison with most of the earlier studies periodicities should be expressed in the form
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\[ A_n \sin (nt + \alpha_n) \]

(II, lb), and the harmonic coefficients \( a_n \) and \( b_n \) (II, la) should also be given since they are often needed for further work.

(2) In all cases where tidal determinations are made, statistical measures of their reliability, such as the radius of the probable error circle should be included (II, 4). If the data are not sufficient to compute such statistical measures they will, as a rule, be entirely unreliable.

(3) To determine \( S_1 \) and \( S_2 \) in the stratosphere and upper troposphere at a given station at least five daily ascents, preferably spaced at equal time intervals, should be made. If higher harmonics, say up to \( n \) (integer), are desired the minimum number of ascents per day is \( 2n + 1 \), equal to the number of harmonic coefficients: two for each harmonic and one for the mean of all observations. The series of ascents should be extended over a sufficiently long period to determine also the seasonal variations.

Since little is known about the magnitude of \( S_1 \) and \( S_2 \) and about the magnitude of the superimposed aperiodic variations it is impossible to predict the length of the time required for satisfactory determinations. It varies presumably from station to station, with elevation, and with the season. After a series has been started one may compute \( S_1 \) and \( S_2 \) for each day, and the radius \( r \) of the probable error circle as more daily values of \( S_1 \) and \( S_2 \) have accumulated (see Section II). After \( r \) has become reasonably small compared to the amplitude the determination may be regarded as successful. According to error theory \( r \) decreases as \( N^{-\frac{1}{2}} \) where \( N \) is the number of determinations. Thus in order to reduce \( r \) to half its value one needs four times as many determinations, that is a four times longer series. In this manner it will be possible to estimate even from a fairly short series, having a large \( r \) how much longer the observation programme should be continued.

(4) Because of the variations of the tidal oscillations with longitude and especially with latitude it will be necessary to have such observations from some widely different locations. For \( S_2 \) in particular it would be important to determine whether the latitude dependence at higher altitudes is similar to that near the ground and whether the standing oscillation (Section III) caused by the distribution of water and land extends to high levels.

In the case of \( S_1 \) little can be conjectured from its irregular distribution at the ground about its distribution at higher altitudes. The distribution away from the ground may be quite regular.

(5) The intermediate layer of the atmosphere, from about 30 to 80 km above the ceiling of conventional balloon flights and below meteor heights is accessible to meteorological rockets. To conduct a satisfactory observational study of the tides in this region rocket ascents of the same distribution in time and space will be required as under (3) and (4), although with the increase of the amplitudes upward the time required for satisfactory determinations may be shorter than in the lower layers.

Since such a programme of rocket ascents at numerous stations will become quite expensive one may inquire if less costly programmes can be devised. If the tides progressed regularly around the earth it would be possible to obtain the desired information by spacing the stations equally along latitude circles and by launching the rockets at the same Greenwich time (or making allowances for unequal spacing in the launching times). However, this procedure will not work once there are irregularities in the progress of the tides. Another, somewhat more promising method would be to change for each station the launching time.
systematically from one firing date to the next. This method would actually not reduce the total number of rocket firings, but only spread it out over a larger time. But it is clearly superior, with respect to the determination of $S_1$ and $S_2$, to a programme in which the launchings occur always at the same time. The lunar tide can, of course, be determined from data taken always at the same hour once a sufficiently long series has been obtained, because this hour corresponds to varying lunar hours.

(6) Because of the large expense of rocket and balloon programmes for determination of the atmospheric tides it would be very desirable to develop some ground-based probing systems based on the propagation of electromagnetic signals through the atmosphere which can measure some meteorological parameter, such as the density, to confirm the results obtained by the other methods, and allow an extension to other times and locations.

(7) The observations of radiometers should be extended over larger time periods to obtain more reliable determinations of the seasonal variations and to permit a determination of the lunar tides. Here, too, more stations spread over both hemispheres, over different latitudes and longitudes are required to find the relative magnitudes of $S_1$ and $S_2$ and their dependence on latitude and longitude.

(8) For still higher levels indirect data on tidal oscillations are available in the form of variations of geomagnetic and ionospheric parameters. But more theoretical work is required before these parameters can be reliably interpreted in terms of such meteorological variables as wind, pressure and density. Because of the world-wide extent of the atmospheric tides a broad international programme is needed to gather the required observations from all parts of the globe.

After sufficient data of the type outlined here have been accumulated, and after tidal theory has been further developed to account for this additional information it will be possible to construct tide tables or graphs similar to those computed by Stolov (1955) for $S_2$ at the earth's surface. This type of information either in tabular or graphical form is clearly needed in the upper atmosphere for a satisfactory interpretation of the individual wind or pressure observation in terms of the atmospheric circulation.

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